Introduction

A Parent's Journal

This book is an example of the literary genre known as the "parent's journal." Children grow and many things happen to them, and their parents diligently record this in a notebook. Years later this proves to be absolutely gripping reading. Especially for the parents. No wonder, this is about their own children, and at such a tender age! And indirectly, it is about their own youth.

How often, dear reader, have you had to scan photos of children; not your own, but others? Guests arrive, pull out a stack of snapshots, and there you go! By the end of the second dozen you try to stifle your yawns but they want to go on to the bitter end. They find everything concerning their children and grandchildren irresistibly fascinating, while your only thought is how all the kids look alike!

Today, presenting this book before you, my feelings are very mixed. I would like to be an objective and impartial observer — but I can't. My wife, Alla Yarkho, is usually a severe and demanding critic. But each time she gets hold of this text, she starts rereading it again and again and is unable to put it aside. Our normal ironic attitude evaporates, replaced by altogether different emotions. This book is about our children; our son, Dima, and our daughter, Jane. The story is about their early years but it is also about their friends. However, we were undeniably much more attentive and observant towards our own children than towards others. This author must answer the harsh question: why should this book be interesting for an outsider? Why should it be of any interest, not only for our family, but for others as well?

I suppose that the answer is that this is not merely a journal, but a journal of a mathematical circle. When Dima was 3 years and 10 months I gathered three of his pals of roughly the same age (or a bit older) and started a mathematical circle. This book describes that unusual experience, and it can also be read as an exercise book of mathematical puzzles for pre-school children. But it also contains descriptions of the children's reactions, what they did and did not understand, and what kind of psychological problems and misunderstandings I encountered in the process.

Most likely, if someone wrote a book containing algebra or geometry problems, and accompanied each problem with a true story of how the students wrestled with it, it would make interesting reading. But with young children, it becomes incomparably more interesting. The evolution of their reasoning, the paths and turns taken by their intellect are much more visible, more vivid. When I started the circle I had no idea how gripping and absorbing that would be. As a result my reading for many years to come was largely composed of books on teaching and psychology; later it broadened to include subjects like linguistics, psychiatry, ethology, and behavioral genetics. I discovered a vast new world; I became richer and, hopefully, wiser — all this thanks to my children.

Pre-school mathematics is relatively simple and thus is accessible to any reader, not just specialists. This makes the potential book audience quite large: teachers, parents, grandparents, math lovers, psychologists, philosophers — you name it. Well, I've managed to convince myself that my book is interesting not just to me. Now I can, with clean conscience, recommend it to you, dear reader. (I must confess however, without false modesty, that many friends who read the manuscript version of my journal have urged me for some time to edit it into book form. Numerous photocopies of the manuscript circulated first in Moscow and later all over the world. This interest has not diminished in the past 20 years; yet another argument for me to create this book.)

Why a circle? Why a journal?

A circle, like any other form of systematic activity, is a way of imposing self-discipline. It's possible to work with children without any circle: now and then you ask questions, assign problems, and have discussions. But in the real world, this almost never works, and soon the whole project evaporates: Yesterday you were too busy, today you are not in the mood or too tired. And why does it have to be this very minute? I will certainly have time later, someday... As a result, nothing much gets accomplished.

But if you know that tomorrow at eleven sharp you will welcome four kids and will have to entertain them for thirty minutes, the situation radically changes. Whether you want to or not, you have to sit somewhere secluded and try to think of something. If you don't have any ideas you will have to borrow them from books. And when you think about something persistently, sooner or later you will have new ideas, which you would never have had had you not been duty-bound.

Or perhaps you get a new idea, but it requires preparation: you have to draw, or use scissors or to glue. To do that, you need more specific obligations than just a noble aspiration to do something "for the sake of the family."

Incidentally, kids, too, enjoy rituals in their play. If adults start interfering with their games out of the blue and pestering them with puzzles, they

will get upset and will wish to get rid of this untimely disturbance as soon as possible. But it's completely different if they know that once a week, on a special day and at a special time we all get together to deal with serious things.

That was a short answer to the question, "Why a circle?"

As for the journal, I did not keep it right from the beginning, and, above all, I did not attach any great importance to our sessions. You will see that the journal starts with the 21st session. The first "20 weeks" do not represent five months, as one could think, but twice as many, because of summer vacation and other periods without sessions. You shouldn't think that because we had decided to have our lessons regularly, once a week, that we actually followed this decision to the letter.

But one day, about six months after the start, some of my friends asked me to tell them what we had been doing. I was happy to open my mouth to talk, but instead of a flood of ideas and puzzles, I paused awkwardly, discovering that I really remembered very few details. What I did remember was the general feeling of enthusiasm and overflow of kids' energy that I felt throughout our sessions. But the specific details seemed to have been lost somewhere along the way. (I felt somewhat like a person who lost his leg: he feels his leg while it no longer exists.)

I did not expect such a dirty trick from my own memory: I was upset and embarrassed. After this meeting I put down what little I still remembered, and decided that from then on, I would write some sort of summary at the end of each session. This way I would at least have a list of puzzles from which I would then be able to reconstruct the rest of the session. It is curious that already at that moment I was thinking about the rest of the session; apparently I felt intuitively how insufficient the mere list of puzzles was going to be. Very soon I made my first "pedagogical discovery:" I found that a record of just the puzzles wasn't very meaningful. It is not the puzzles, nor their solutions, that are interesting, but the process, the path that connects them. Mathematical problems for preschool kids are, objectively, trivial (inventing them, however, is not trivial). On the other hand, it can take a child several years to solve a puzzle that somebody sets him. I mean it — several years! You will see many examples. During all this time the child's mind never rests: pacing and racing, like one possessed, alert to anything that may pertain to the puzzles. This process of solution is the most interesting thing of all.

Thus my list of puzzles gradually added more and more comments, narratives, and anecdotes, and began to include not only mathematical remarks but those of a more general and sometimes "theoretical" nature, finally resulting in this journal. The next stage was feedback between the journal and the circle. When you put to paper your ideas and your thoughts, they give rise to new ideas, new points of view, new puzzles and topics. Your understanding deepens. Even the capacity to observe seems to be sharpened. Sometimes you recall something that went unnoticed in the excitement of

the session, and you could have easily forgotten it, had it not been noted. This symbiosis between the circle and the journal worked grew so that it became hard to imagine one without the other.

Last but not least: My children, now grown, have read my journal. They remember many things well, but sometimes their version completely contradicts mine. At my request they added their commentaries which enriched the whole project with a new dimension. One of my friends said that it gave the journal a stereoscopic effect.

Should journals be edited?

I think — yes, absolutely. Long ago, some authors said, "These pages will never be cheapened by the printing press." This was usually insincere: they secretly believed that thrilled, grateful descendants would find their work, would be enraptured by their modesty and sincerity ("imagine that it was never intended to be made public!"), would publish it and that would bring them eternal glory. It is really simple to catch such an author off-guard: he endlessly explains to himself what he would seem to know perfectly well. As a matter of fact, it is the reader who needs all these explanations, not the author.

If, however, the journal was truly intended for the author only and then someone else read it, it could lead to monstrous distortions. Imagine the following situation: there is a person who has my infinite respect, so great that sometimes it would literally come close to worship. However, I am not blind to certain small (or not so small) flaws or funny traits of his character. In the journal "for myself" I can permit myself an ironic, mocking or even an irritated remark, without necessarily reminding myself of my great respect towards this person. ("Only don't think I do not have immense respect for him" ... don't think — who?) But if an outsider would happen to read this text later, he might misinterpret it, seeing only the sarcasm and not knowing about my respect.

Here is another, more pertinent example. I am writing about kids, or so it seems. In fact, I am only writing about one aspect of their life: namely, their relationship with mathematics. In fact, I am describing half an hour out of every two weeks of their life; it would be crazy to think that a child can be reduced in this way. Each of them has numerous interests, numerous problems, and many other talents. But when I myself read my journal, I sometimes forget about this, so it's only natural that a reader who has never met the kids personally, would understand so little about them. (I develop this idea later, but only concerning my daughter — see p. 241.) Not to mention the fact that the former kids have long ago turned into adults. Soon that will be their own kids who will read the book about their moms and dads. The journal that once pretended to be a documentary would more and more become fiction.

That would not be true to say that my journal was a strictly intimate document. It was rather a technical thing. I wrote it for myself but its contents were never secret and I was willingly sharing it with all those interested. But its technical nature created its own problems. I would not explain for my own sake the problem of the wolf, the goat, and the cabbage, or describe in detail the Tower of Hanoi.[1] Also, the original text had many repetitions, notes that needed decoding or additional explanations. (I left some repetitions as they are, especially in cases when on the basis of the same premises I came to quite different conclusions — this way it looks more authentic.)

In short, there are many reasons, both ethical and technical, why my journal needed very serious editing. That's why for many years I would refuse all suggestions to publish it "as is."

Why then it took me so long to edit it? What was I doing all this time?

Well, life is life. For starters.

Anyway, I think it's time to stop beating about the bush and get down to business.

A novice's reflections on pre-school math

When does it begin?

All of you must have seen such scenes many times: a young mother hides behind a curtain, then emerges, smiling, with a "Peek-a-boo," then disappears again. Her toddler greets each of these appearances by clapping his hands and shrieking with glee. Both are utterly happy. And both have no idea that what they are doing is mathematics.

The last phrase is not there merely to shock the reader or to stun him with a far-fetched paradox. I mean it. According to psychologists, up to the age of eighteen months, the most important intellectual problem which a child faces is to discover the law of object permanence. This means that the objects out of sight have not completely disappeared; they still exist somewhere, although we can't see them. The child discovers that such an important object as "mother," even if she has disappeared behind the curtain, is still there and will soon reappear from behind that same curtain.

As a child grows, so does his understanding of the world. A very small girl is playing an exciting game: she picks up blocks scattered on the floor and hands them, one by one, to her dad accompanying each gift with a jubilant cry: "Here!" Her Dad takes each block and she laughs happily. She has learned the word "Here" only recently and tries to use it as much as possible. But suddenly her little clumsy hands seize two blocks at once. She thinks for a few moments and then — "Eureka" — she offers them to her dad saying: "Here-here!"

[1] *P. Z.'s Note:* These are two classic "folklore" problems, see p. 43 and p. 286.

To paraphrase Pushkin, "It is a most fascinating pastime to follow a little one's thoughts." [1] By the age of two many things have already been mastered. A boy of two tries to wake up his dad in the morning:

"Papa, are you asleep?"

"No I am not," answers his dad rubbing his eyes. "I am in the kitchen, having tea."

His son is bewildered: this contradicts everything he knows. Just in case he runs off to the kitchen to check. His return is triumphant:

"No, you are *not* in the kitchen. You're here, you're here!"

Next time he won't be fooled that easily. But I would like to insist on this moment of independent research when he ran off to the kitchen to check. Quite intuitively I feel it to be a very important quality and wish children to preserve it as long as possible.

Counting in Japanese

Somewhat later the kid will be taught math "for real." As a rule it means that s/he will have to learn how to count. Certainly, it is important to know how to count. But adults don't always fully realize what is at stake here.

Let's try to get into the child's shoes and learn to count, but in Japanese. Here are the first ten numbers: *ichi, ni, san, shi, go, roku, shichi, hachi, kyu, ju.* The first assignment is to learn this sequence by heart. You will see: it is not that easy! When you have managed to do that, pass on to the second assignment: learn how to count backwards, from *ju* to *ichi.* Once you have done that, try and calculate. How much is *roku* and *san*? *Shichi* minus *go*? *Hachi* divided by *shi*? And now a sum for you: Mother bought at the market *kyu* apples and gave each of *shi* children *ni* apples. How many apples are left? There is just one rule: you are not allowed to translate into English, even mentally. After a bit of practice, it becomes involuntary. Sometimes we are not even aware of it.

I was able to appreciate the intellectual feat of children learning how to count only much later, when I came to live in France. I have been living here for more than 15 years now, but I still have trouble with French numerals. The French have a peculiar system for the names of the numbers between 70 and 99. After sixty-nine they have sixty-ten (70), then sixty-eleven (71), sixty-twelve (72), etc.; after sixty-nineteen there appears four-twenties (80), four-twenties-one (81), four-twenties-two (82)... and after four-twenties-nine (89) there comes again four-twenties-ten (90), four-twenties-eleven (91), four-twenties-twelve (92)... four-twenties-nineteen (99) and only then one hundred. When someone gives me a telephone number quickly or the dates of a famous person, believe me, it is not easy to catch. Fortunately, I do not have to add or to subtract.

[1] The great Russian poet Pushkin once said, "It is a most fascinating pastime to follow a great man's thoughts."

(By the way, this is the origin of the funny answer of a French elementary school pupil, often cited in the teaching literature: when asked, "How much is twenty times four," he answered "Four-twenties, because multiplication is commutative.")

At last you can fluently count up to *ju*. How long did it take you? A week? A month? Now you are aware of the fact this is not merely a problem of rote learning — if it had been the case, you would have made it in an hour. But if not rote learning, then what? Are you able to isolate from your experience purely mathematical problems which are always present in counting but remain hidden, unnoticed? Not that easy, is it?

But perhaps it's all for the better. Otherwise enthusiasts of early teaching, in their hurry to pull a kid up the stairs, would have rushed to explain to him what he is not yet able to understand. And he could have arrived there on his own.

Nursery school geometry

Along with counting comes basic geometry. Traditionally, it is thought that kids should learn (a bit) about geometrical figures and shapes such as triangles, squares, circles, etc., and also simple measuring. But think: if a child quite easily tells a fork from a spoon, why it should ever be difficult for him to tell a square from a triangle? In fact, this is not at all difficult! The really difficult point is understanding logical relationships and what actions can be perfomed on geometric shapes. For instance, many elementary school pupils believe that a square put sideways is no longer a square but a quadrilateral (Figure 1). On the other hand, for example, understanding the question, "What is there more of, squares or quadrilaterals?" requires exceptional logic.

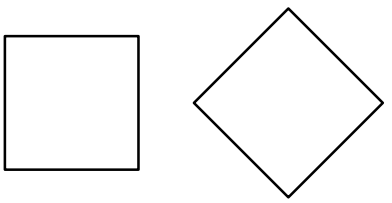

Figure 1. The left figure is a square. And the right one? Kids often think it is not: in their eyes a turned square loses the status of a square and becomes merely a quadrilateral.

From this point of view squares and triangles immediately lose their primacy: puzzles with forks and spoons as objects are no less mathematical if there is food for thought. To put it otherwise, school mathematics deals with "numbers and shapes" and this is how it should be. However, we cannot tell kids much about these objects. This, in principle, could mean that we cannot deliver to kids any worthy mathematical information. As a matter of fact, this is not true. Actually we have plenty of interesting things to show them, but we have to be careful to develop the right approach.

But what is the right approach? Well, there are as many opinions as there are people. Below I cite some of them (sometimes exaggerating, to make my point of view clearer).

Opinions

(1) *Our contemporary high-tech society requires children to be exposed to much more mathematics, even before formal schooling begins.*

Too boring! To hell with all these requirements!

(2) *It's dangerous, you know, to stuff a child's brain with such complicated stuff. All these integrals and the like!*

My God, who mentioned integrals?

(3) *Can you imagine this?! He makes little kids study probability theory! Adults with a higher education fail to understand, and kids cope perfectly well! I always said that the potential of our brain remains unknown, especially in early childhood.*

Dear enthusiastic observer, I am afraid you are mistaken: we do not study probability theory though we do observe certain probabilistic phenomena. (A person at a bus stop trying to guess which bus will come first is doing the same.)

(4) *At an early age, children usually have an excellent memory and capacity to learn new things. Furthermore, they really pay attention to what adults say. Therefore, they should be fed as much information as possible. Later, when their critical thinking skills improve, and they will not want to do things that do not make sense for them, we should give them more time for reflection and add motivation to their learning.*

I got this point of view from the internet. The author is a university professor, a neurophysiologist. Since I do not divulge his name, permit me to frankly express my point of view: this crap does not deserve a response.

(5) *I don't see why we should stuff kids' heads with this nonsense. Leave them alone; let them have a normal childhood!*

Here my opponent has raised two issues: one concerns "nonsense," the other,"normal childhood." I won't argue about nonsense; after all, this is a matter of taste. But I was doing with kids what I enjoy doing myself, and this aspect of our lessons seems to me very important.

When my articles appeared, I, quite unexpectedly, found myself in a role totally alien to my nature, namely, as a giver of advice. I got letters from parents. One of them (a mother) vividly wrote, "I've hated math since childhood. But I know that it's very important for a child's intellectual development. Could you advise me on how to do math with my son?" Fortunately I knew how I should reply in this case. I told her, "Don't do any math with your son if you hate it. Do with him only what you yourself enjoy doing, and then your engagement will be a source of joy for both of you. If you like to bake, say, do this with your son..." The only thing that worried me was that she might feel insulted by my advice; she might think that I considered her not smart enough to do math.

> But I do have a quarrel with the idea of a "normal childhood." Imagine the following scene. We are sitting on the bank of a river watching a solitary wasp who makes a hole in the packed sand. When it finishes the hole, it will fly away and soon come back with enough food for her progeny, e.g., paralyzed spiders, then it will lay eggs inside the prey and close the hole. The famous biologist and ethologist Nikolaas Tinbergen demonstrated that wasps navigate by landmarks: we can put a shoe to the right of the nest and a shell to the left, and when the wasp flies away, we can move both a meter aside. A couple of minutes later the wasp comes back and, exactly as it has been discovered by Tinbergen, it lands not near the nest but a meter aside, between the shoe and the shell. This experience was welcomed with a lot of enthusiasm not only by our children but also by other kids who were on the beach. (An interesting observation is that it occurred to none of them to squash the wasp.) Well, does this sort of pastime correspond to your idea of a normal childhood? It gave Tinbergen his Nobel Prize, which could provoke enthusiastic observer #3 above to say something pompously foolish about prodigies carrying out Nobel-Prize experiments.

If I taught kids anything at all, it was the idea that they should experience their world with openness and interest. I will always remember the words of my close friend Andrei Toom, the remarkable mathematician and teacher. I cite them here not to compliment me, but rather as a beautiful description of an ideal of teaching to which one must aspire:

What you teach them is not mathematics but a way of life.

A Short History of Our Circle

I found the date of our first session quite by chance: March 23, 1980. Our lessons with the boys went on for four years, with some regularity. By then, my daughter was older and I started a second circle, with her and her friends, which went on for two years. There were a few other attempts to launch circles with other children, but none of them lasted long. I did all that side by side with my permanent job (although my heart was not in it; see p. 11). A few times I even performed as a visiting "maestro" giving a master-class.

At some point, however, I mustered up my courage and went to teach in schools. First I led a circle at Dima's elementary school; then a few similar experiences, and then I did something completely crazy: threw myself headlong into full-time teaching in a Moscow elementary school (testing an experimental curriculum, and only for a month). Previously, I was an arrogant intellectual, always ready to criticize schools and teachers and give them sage advice. But this cruel experiment on myself really opened my eyes.

But I am getting ahead of things.

An important step in this story occurred when a friend of our family, the well-known psycholinguist Revekka Frumkina (whose home workshops I had attended),[1] having read my journal, took me to the editorial office of the magazine *Znanie — Sila* (Knowledge is Power) that soon afterwards published my two papers about the circle (no. 8, 1985, and no. 2, 1986). There was also a third paper but it did not reach the sudden popularity of the first two. A remarkable children's poet and teacher, Vadim Levin, even told me once that he considered them as classics of teaching literature. Here is what happened to them afterwards: some time later both were published in English in *The Journal of Mathematical Behavior* (no. 2, 1992, and no. 2, 1993); then were published on the web on four different sites, if I am not mistaken; then were reprinted in the Russian journal *Pre-school Education* (May and July 2000), this time with commentaries by Vadim Levin; later they became part of his book *Lessons for Parents* (Moscow, Folio, 2001). Quite recently the September First publishing house came out with a booklet entitled *Home School for Kids* with the same text. Unfortunately, in some publications (especially online), the name of Levin is missing. In this way some compliments by Levin to me seem to be made by myself. It is hard to rectify the situation.

Meanwhile, it was the beginning of *perestroika*.[2] Our close friend Stepan Pachikov organized in Moscow a computer club for children; the computers were offered by the World Chess Champion Garry Kasparov. At that time (1986) it was probably the only place in the Soviet Union where schoolchildren had access to computers. I was, predictably, entrusted with a group of the youngest kids. Then followed summer computer camps in Pereslavl'-Zalessky and a winter camp in Zvenigorod. There was another computer club for children, "Zodiac." It is hard to recall everything. I was very much in demand and everybody considered me an expert on pre-school math education. Eventually, I got involved with the so-called "Temporary Scientific Team School-1" predestined to prepare a reform of the entire school system; its formal head was the academician Velikhov, but its actual leader was Alexei Semenov. Among our many activities were the textbook *Algorithmica* for forms 5–7 (ages 12–15); its authors, in Russian alphabetic order, are A. K. Zvonkin, A. G. Kulakov, S. K. Lando, A. L. Semenov, A. K. Shen'; after several mimeographed publications, the Drofa publishing house came out with several editions, starting in 1986.[3] Some of the subjects that first

[1] *P. Z.'s Note:* The "home workshop" was a Soviet-era gathering of intellectuals at a participant's home. These gatherings did serious work, but without incurring official scrutiny.

[2] *P. Z.'s Note:* The reforms, begun by Gorbachev, which eventually led to the dissolution of the Soviet Union.

[3] The latest edition came out in 2006: A. K. Zvonkin, S. K. Lando, A. L. Semenov, *Informatica, Algorithmica: Textbook for 6th-year students*, Moscow, Prosveshchenie (publisher); "Education."

appeared at our circle were in a slightly more complicated form than that used in the textbook. I was amused to learn that the course was chosen for teaching freshmen at an American university.

...

Then my life took a sudden turn, which relates to the circle, albeit indirectly. In 1989 I changed my job after 13 years in the "All-Union Research and Development Institute for Automation of the Gas and Oil Industry." Those who lived through the Soviet period know about these places: third-rate institutes of applied research, with guardians monitoring attendance, and annual missions to a *kolkhoz*[1] to help with their work; cordial and helpful relationships with like-suffering colleagues; extremely dull and useless work (nowadays the same people do quite meaningful and practical things but elsewhere). I hated my job but there was no chance I could ever find another one. And all of a sudden I found myself in the Scientific Council of Cybernetics of the USSR Academy of Sciences, in a team introducing computer science into school curricula. I think our results were far from brilliant but at that time we were full of enthusiasm and so carried away by our work that sometimes we were literally on the verge of physical exhaustion. Ignoring the larger forces of history, in terms of my life story it is clear that without the circle I would never have gotten this wonderful new job.

I still continued mathematical research. That is, side by side with research in teaching I was also doing scientific research in the domain of mathematics. But I radically changed my direction of research. Among other things, this change was stimulated by the fact that in the above-mentioned computer camp in Pereslavl'-Zalessky I found two colleagues equally enthusiastic about my new mathematical interests: Sergei Lando and George Shabat. We started working together and are still going strong: our joint monograph with Sergei Lando was recently (2004) published by Springer-Verlag, and the Russian translation appeared in 2010.

In 1990 I first visited France. Had I still worked in my former institute, there would be no question of traveling abroad. In the Council of Cybernetics the situation was utterly different. At that time I did not yet know my colleagues in my new research field. This trip got me oriented; I found people with similar interests working in Bordeaux. I went there once, then again; then I was invited as a visiting professor for a year. Eventually, I stayed on for good as a full professor. It's a strange turn of events, that the fact that I am a professor in Bordeaux today is largely due to my starting a mathematical circle for kids years ago.

[1] *P. Z.'s Note:* Collective farm. Zvonkin is referring to regular "volunteer" work solicited by the government.

Acknowledgments

My readers, intelligent folks, can easily guess the first people I will thank: the children. When I was young my parents would often pester me with, "You'll understand when you have kids of your own." That angered me: What is so special to understand?! Kids of your own — big deal! Everybody has kids, so what?

How could I know at that time that having a child brings about a completely new worldview? I am sure that some people, with more vivid imagination, can intuit this without having children of their own; for such people, when they do have kids, the changes may be less violent. But in my personal universe, my children were the Big Bang, and I thank them for it.

I am profoundly grateful to all the children who participated in our circles. It is hard to imagine how much I have learned from them. But I wish to tell them one thing. Quite honestly and as best as I could, I tried to share my attention equally among them — but I am afraid I did not always succeed. I hope I did not hurt anybody. However, if any of them has any lasting grudges, I can only say, "You'll understand when you have kids of your own."

I am grateful to my wife Alla Yarkho. First, because we had children: her part of the job is hard to overestimate. Second, because it's her who thought of the circle. You will see that in fact the whole thing was as much hers as mine. I am grateful for her moral support as well as her later insistence that I should prepare the journal for publishing. It goes without saying that throughout all these years she has been a proofreader, an editor, an adviser and simply a passionate reader.

Revekka Markovna Frumkina "launched" me. Thanks to her our family enterprise became popular. Rita Markovna (as her friends call her) is a godmother of numerous interesting projects. This is one of them. And our discussions helped to clarify my ideas.

Alexei Semenov was not only my official supervisor in the council of Cybernetics, but also served as an "elder advisor" (though he is younger than me). And his conviction that my work deserved a larger audience helped persuade me.

Alexander Shen' was one of the early readers of the handwritten notebooks and often urged me to publish them. Eventually, tired of my foot-dragging, he assembled a team of high school and college students to typeset the first version of this book. Vladimir Lugovkin, particularly, did an enormous job. At this point, I had no choice but to edit the book. And again the indispensable Shen' helped me by installing the Russian version of LaTeX onto my French computer, saving me years of work.

The list of acknowledgments is far from being finished — indeed, I did not even begin properly. But if I am going to continue this way, the reader will surely skip this part. So I will have to be brief: I am profoundly grateful to

all those with whom I ever discussed this book, be these discussions long or short, who encouraged me with their interest. Thank you all, dear friends!

. . .

In Moscow, on Bolshoi Vlassievskii Lane (near Arbat Street) stands a building which houses a remarkable institution that has already become widely popular among mathematicians all over the world. It is called The Moscow Center for Continuous Mathematical Education.[1] "Continuous" here means "for all ages." It comprises a publishing house, the Independent University of Moscow and its post-graduate school, and also organizes numerous mathematical Olympiads and competitions for middle and high school. So why not include pre-school children? I am happy and proud that my book was published by MCCME. Besides, in a place like MCCME, one is bound to meet old friends. One of them, Vadim Bugaenko, was responsible for the initial stages of editing this book, even though it had never been his direct duty. Later his help also proved inestimable when we worked on the second and third Russian editions, as well as the English one that you are reading now. Readers often tend to credit the author with all the glory. They sometimes don't realize how complicated and multifaceted the process of editing can be.

I am grateful to the entire team but first and foremost to Mikhail Panov who, among other things, made all the drawings. (Experts will be able to appreciate his virtuoso skills to make real pictures by means of the METAPOST software initially conceived as a tool for drawing graphic images for scientific papers.)

Last but not least, I must note the outstanding role played by my American colleagues Paul Zeitz and Silvio Levy, as well as by the entire publications department of MSRI. All the gratitude mentioned above to my Russian helpers applies equally to them. Finally, I'd like to note the important role played by the American Mathematical Society (AMS) in initiating an American edition of this book.

Two Disclaimers

First: I did not create all of the puzzles used in this book. I borrowed them from many different sources, including psychology and recreational math books, and friends. I preserved some in their original form, and altered others, often beyond recognition, during the course of the sessions. I forgot some of my sources long ago; others were lost when I moved to France. For some problems I do not even remember whether they are borrowed or mine. To pay back this debt to the world, I allow everybody to use them and

[1] See www.mccme.ru. This website is in Russian, but substantial portions are translated into English.

pledge not to sue anybody for plagiarism. On the contrary I will be glad (and even proud) if my inventions become part of teaching folklore.

Second: my stories are not stenographic reports. They were noted from memory, and it is not always easy to discern what kids say when they all talk at the same time. I am sure to have missed or misinterpreted some things. But I believe this is more or less obvious.