Abstract

We introduce a class of multilinear singular integral forms

$$\Lambda : L^{p_1}(\mathbb{R}^d) \times \cdots \times L^{p_{n+2}}(\mathbb{R}^d) \to \mathbb{C}$$

which generalize the Christ-Journé multilinear forms; here $\sum_{j=1}^{n+2} p_j^{-1} = 1$, $p_j \in (1, \infty]$. The research is partially motivated by an approach to Bressan's problem on incompressible mixing flows. A key aspect of the theory is that the class of operators is closed under adjoints (i.e. the class of multilinear forms is closed under permutations of the entries). This, together with an interpolation, allows us to reduce the $L^{p_1} \times \cdots \times L^{p_{n+2}}$ boundedness to $L^\infty \times \cdots \times L^\infty \times L^p \times L^{p'}$ boundedness. We obtain estimates of the form

$$|\Lambda(f_1, \ldots, f_{n+2})| \leq C n^2 \log^3(2+n) \prod_{j=1}^{n+2} \|f_j\|_{L^{p_j}},$$

where the constant C does not depend on n.