Abstract

A set V in a domain U in $\mathbb{C}^n$ has the *norm-preserving extension property* if every bounded holomorphic function on V has a holomorphic extension to U with the same supremum norm. We prove that an algebraic subset of the *symmetrized bidisc*

$$G \stackrel{\text{def}}{=} \{(z+w, zw) : |z| < 1,\ |w| < 1\}$$

has the norm-preserving extension property if and only if it is either a singleton, G itself, a complex geodesic of G, or the union of the set $\{(2z, z^2) : |z| < 1\}$ and a complex geodesic of degree 1 in G. We also prove that the complex geodesics in G coincide with the nontrivial holomorphic retracts in G. Thus, in contrast to the case of the ball or the bidisc, there are sets in G which have the norm-preserving extension property but are not holomorphic retracts of G. In the course of the proof we obtain a detailed classification of the complex geodesics in G modulo automorphisms of G. We give applications to von Neumann-type inequalities for Γ-contractions (that is, commuting pairs of operators for which the closure of G is a spectral set) and for symmetric functions of commuting pairs of contractive operators. We find three other domains that contain sets with the norm-preserving extension property which are not retracts: they are the spectral ball of 2×2 matrices, the tetrablock and the pentablock. We also identify the subsets of the bidisc which have the norm-preserving extension property for symmetric functions.