

Contents

Chapter 1. Introduction	1
1.1. Background	1
1.2. The goal: affine type C	3
1.3. An overview	6
1.4. The organization	9
Part 1. Affine flag varieties, Schur algebras, and Lusztig algebras	13
Chapter 2. Constructions in affine type A	15
2.1. Lattice presentation of affine flag varieties of type A	15
2.2. Monomial basis for quantum affine \mathfrak{sl}_n	16
2.3. Algebras \mathbf{U}_n and $\dot{\mathbf{U}}_n$	20
Chapter 3. Lattice presentation of affine flag varieties of type C	23
3.1. Affine complete flag varieties of type C	23
3.2. Affine partial flag varieties of type C	25
3.3. Local property at L_0	26
Chapter 4. Multiplication formulas for Chevalley generators	29
4.1. Some dimension computation	29
4.2. Standard and canonical bases of Schur algebras	30
4.3. Some multiplication formulas	33
4.4. The leading term	37
Chapter 5. Coideal algebra type structures of Schur algebras and Lusztig algebras	41
5.1. The Lusztig algebra $\mathbf{U}_{n,d}^c$	41
5.2. A raw comultiplication	43
5.3. The comultiplication Δ^c	44
5.4. Monomial and canonical bases of $\mathbf{U}_{n,d}^c$	49
Part 2. Lusztig algebras and coideal subalgebras of $\mathbf{U}(\widehat{\mathfrak{sl}}_n)$	51
Chapter 6. Realization of the idempotent coideal subalgebra $\dot{\mathbf{U}}_n^c$ of $\mathbf{U}(\widehat{\mathfrak{sl}}_n)$	53
6.1. The coideal subalgebra \mathbf{U}_n^c of \mathbf{U}_n	53
6.2. The algebra $\dot{\mathbf{U}}_n^c$ and its monomial basis	56
6.3. Bilinear form on $\dot{\mathbf{U}}_n^c$	58
6.4. The canonical basis of $\dot{\mathbf{U}}_n^c$ and positivity	59
6.5. Another presentation of the algebra $\dot{\mathbf{U}}_n^c$	60

Chapter 7. A second coideal subalgebra of quantum affine \mathfrak{sl}_n	63
7.1. The Schur algebras of type \mathfrak{sl}_n	63
7.2. The comultiplication	64
7.3. The monomial basis of $\mathbf{U}_{n,d}^{\mathfrak{sl}_n}$	67
7.4. The coideal subalgebra of type \mathfrak{sl}_n	69
Chapter 8. More variants of coideal subalgebras of quantum affine \mathfrak{sl}_n	73
8.1. The Schur algebras of type \mathfrak{sl}_n	73
8.2. Comultiplication and transfer map of type \mathfrak{sl}_n	74
8.3. Quantum symmetric pair $(\mathbf{U}(\widehat{\mathfrak{sl}}_n), \mathbf{U}_n^{\mathfrak{sl}_n})$ and canonical basis on $\dot{\mathbf{U}}_n^{\mathfrak{sl}_n}$	76
8.4. The Schur algebras of type \mathfrak{sl}_n	77
8.5. Realization of a new coideal subalgebra $\mathbf{U}_\eta^{\mathfrak{sl}_n}$	79
Part 3. Schur algebras and coideal subalgebras of $\mathbf{U}(\widehat{\mathfrak{gl}}_n)$	83
Chapter 9. The stabilization algebra $\dot{\mathbf{K}}_n^c$ arising from Schur algebras	85
9.1. Monomial bases for Schur algebras	85
9.2. Stabilization of the Schur algebras	89
9.3. Comultiplication and stabilization	92
9.4. The algebra $\dot{\mathbf{K}}_n^c$ and its stably canonical basis	93
9.5. The algebra $\dot{\mathbf{K}}_n^c$ of affine type A and its comultiplication	95
9.6. The comultiplication on \mathbf{K}_n^c	98
9.7. A homomorphism from \mathbf{K}_n^c to $\mathbf{S}_{n,d}^c$	99
9.8. The algebra $\dot{\mathbf{K}}_n^c$ as a subquotient of $\dot{\mathbf{K}}_n^c$	100
Chapter 10. Stabilization algebras arising from other Schur algebras	101
10.1. A monomial basis for Schur algebra $\mathbf{S}_{n,d}^{\mathfrak{sl}_n}$	101
10.2. Stabilization of Schur algebras of type \mathfrak{sl}_n	102
10.3. The stabilization algebra $\dot{\mathbf{K}}_n^{\mathfrak{sl}_n}$	103
10.4. Stabilization algebra of type \mathfrak{sl}_n	106
10.5. Stabilization algebra of type \mathfrak{sl}_n	107
Appendix A. Constructions in finite type C	109
A.1. Multiplication formulas	109
A.2. Isomorphisms between type C and type B	112
A.3. The comultiplication	113
Nomenclature	117
Bibliography	121