

Contents

Chapter 1. Introduction	1
1.1. History and context of new results	1
1.2. Linearized equations	4
1.3. Streaks	8
1.4. Statement of result	9
1.5. Discussion of Theorem 1	12
1.6. Notations and Conventions	13
Chapter 2. Outline of the proof	15
2.1. Summary and weakly nonlinear heuristics	15
2.2. Instantaneous regularization and continuation of solutions	19
2.3. Q^i formulation, the coordinate transformation, and some key cancellations	20
2.4. The toy model and design of the norms	25
2.5. Main energy estimates	32
Chapter 3. Regularization and continuation	41
3.1. Regularization and short-time regularity	41
3.2. Moving from (X, Y, Z) to (x, y, z)	43
3.3. Proposition 2.1 implies Theorem 1 and Proposition 1.4	44
Chapter 4. High norm estimate on Q^2	47
4.1. Zero frequencies	48
4.2. Non-zero frequencies	53
Chapter 5. High norm estimate on Q^3	67
5.1. Zero frequencies	68
5.2. Non-zero frequencies	70
Chapter 6. High norm estimate on Q_0^1	83
6.1. Improvement of (2.51a)	83
6.2. Low norm estimate on Q_0^1 , improvement to (2.51b)	88
Chapter 7. High norm estimate on Q_{\neq}^1	89
7.1. Linear stretching term $LS1$	89
7.2. Lift-up effect term LU	91
7.3. Linear pressure term $LP1$	92
7.4. Nonlinear pressure NLP	92
7.5. Nonlinear stretching NLS	92
7.6. Transport nonlinearity \mathcal{T}	94
7.7. Dissipation error terms \mathcal{D}	94

Chapter 8. Coordinate system controls	95
8.1. High norm estimate on g	95
8.2. Low norm estimate on g	96
8.3. Long time, high norm estimate on C : improvement to (2.52a)	97
8.4. Shorter time, high norm estimate on C : improvement to (2.52b)	99
Chapter 9. Enhanced dissipation estimates	101
9.1. Enhanced dissipation of Q^3	101
9.2. Enhanced dissipation of Q^2	106
9.3. Enhanced dissipation of Q^1	110
Chapter 10. Sobolev estimates	113
10.1. Improvement of (2.54a)	113
10.2. Improvement of (2.54c)	115
10.3. Improvement of (2.54b)	116
10.4. Improvement of (2.54d) and (2.54e)	116
10.5. Improvement of (2.54f)	117
10.6. Improvement of (2.54g) and (2.54h)	118
Acknowledgments	119
Appendix A. Fourier analysis conventions, elementary inequalities, and Gevrey spaces	121
Appendix B. Definition and analysis of norms	125
B.1. Definition and analysis of w	125
B.2. The design and analysis of w_L	127
Appendix C. Multiplier and paraproduct tools	129
C.1. Basic inequalities regarding the multipliers	129
C.2. Paraproducts and related notations	136
C.3. Product lemmas and a few immediate consequences	139
Appendix D. Elliptic estimates	143
D.1. Lossy estimates	143
D.2. Precision lemmas	145
Bibliography	151