

Abstract

The most general construction of double affine Artin groups (DAAG) and Hecke algebras (DAHA) associates such objects to pairs of compatible reductive group data. We show that DAAG/DAHA *always* admit a faithful action by automorphisms of a finite index subgroup of the Artin group of type A_2 , which descends to a faithful outer action of a congruence subgroup of $\mathrm{SL}(2, \mathbb{Z})$ or $\mathrm{PSL}(2, \mathbb{Z})$. This was previously known only in some special cases and, to the best of our knowledge, not even conjectured to hold in full generality.

It turns out that the structural intricacies of DAAG/DAHA are captured by the underlying *semisimple* data and, to a large extent, even by *adjoint* data; we prove our main result by reduction to the adjoint case. Adjoint DAAG/DAHA correspond in a natural way to affine Lie algebras, or more precisely to their affinized Weyl groups, which are the semi-direct products $W \ltimes Q^\vee$ of the Weyl group W with the coroot lattice Q^\vee . They were defined topologically by van der Lek, and independently, algebraically, by Cherednik. We now describe our results for the adjoint case in greater detail.

We first give a new Coxeter-type presentation for adjoint DAAG as quotients of the Coxeter braid groups associated to certain crystallographic diagrams that we call double affine Coxeter diagrams. As a consequence we show that the rank two Artin groups of type A_2, B_2, G_2 act by automorphisms on the adjoint DAAG/DAHA associated to affine Lie algebras of twist number $r = 1, 2, 3$, respectively. This extends a fundamental result of Cherednik for $r = 1$.

We show further that the above rank two Artin group action descends to an outer action of the congruence subgroup $\Gamma_1(r)$. In particular, $\Gamma_1(r)$ acts naturally on the set of isomorphism classes of representations of an adjoint DAAG/DAHA of twist number r , giving rise to a projective representation of $\Gamma_1(r)$ on the space of a $\Gamma_1(r)$ -stable representation. We also provide a classification of the involutions of Kazhdan-Lusztig type that appear in the context of these actions.
