Abstract

Let g and $\mathcal{N}$ be arbitrary positive integers and let p be any prime number with $p \nmid \mathcal{N}$. Let $\Gamma_g(\mathcal{N})$ denote the principal congruence subgroup of level $\mathcal{N}$ of $Sp(g,\mathbb{Z})(\subset GL(2g,\mathbb{Z}))$. Let $\mathcal{S}_m(\Gamma_g(\mathcal{N}))$ denote the space of holomorphic Siegel cusp forms of any weight $m \geq g+1$ on $\Gamma_g(\mathcal{N})$. Here we write our main results roughly. We analyse the action of Hecke rings on Siegel modular varieties of arbitrary degrees and arbitrary levels≥ 3 using arithmetic toroidal compactifications and rigid analytic spaces, ℓ-adic cohomology and p-adic Hodge theory. We give new congruence relations for Hecke correspondences on Siegel modular varieties. We express Hecke operators $T_m(p)$ and $T_m(p^2,j)$ with $0 \leq j \leq g$ acting on $\mathcal{S}_m(\Gamma_g(\mathcal{N}))$ by endomorphisms of certain rigid analytic varieties. We give estimation of any Archimedean absolute value of any eigenvalue of $T_m(p)$ and $T_m(p^2,j)$ with $0 \leq j \leq g$ with right proofs. We write the estimate for any eigenvalue of $T_m(p)$ in a form of p-product expansion. We show existence of a Siegel cusp eigenform in every non-zero $\mathfrak{S}_m(\mathcal{N},\chi)$ (for which see Chapter 6 of this paper) $\subset \mathcal{S}_m(\Gamma_g(\mathcal{N}))$ whose p-parameters satisfy $|\alpha_0(p)| = p^{\frac{gm}{2}-\frac{g(g+1)}{4}}$ and $|\alpha_j(p)| = p^{j-\frac{g+1}{2}}$ for any $1 \leq j \leq g$ and any prime $p \nmid \mathcal{N}$. Now fix any integer $g \geq 1$. Let $\{k_j | 1 \leq j \leq g\}$ be arbitrary numbers in $2^{-1}\mathbb{Z}$ with $\sum_{j=1}^{g} k_j = 0$, $0 \leq k_j \leq j - \frac{g+1}{2}$ for any $\frac{g+1}{2} \leq j \leq g$, and $0 \geq k_j \geq j - \frac{g+1}{2}$ for any $1 \leq j < \frac{g+1}{2}$. We show there are infinitely many integers $m \geq g+1$ such that there exists an eigenform $\in \mathcal{S}_m(\Gamma_g(\mathcal{N}))$ whose p-parameters $\{\alpha_j(p) \mid 0 \leq j \leq g\}$ satisfy $|\alpha_0(p)| = p^{\frac{gm}{2}-\frac{g(g+1)}{4}}$ and $|\alpha_j(p)| = p^{k_j}$ for any prime $p \nmid \mathcal{N}$ and any integer $j \in [1,g]$. We can let $k_j = 0$ for all $j \in [1,g]$. Therefore for any integer $g \geq 1$ we have infinitely many holomorphic Siegel cusp eigenforms of degree g that satisfy the generalized Ramanujan conjecture. For any eigenform$\in \mathcal{S}_m(\Gamma_2(\mathcal{N}))$ with $m \geq 3$, we give also estimations of $|\alpha_2(p)|$ and $|\alpha_1(p)|$ for any prime $p \nmid \mathcal{N}$.