Abstract

We investigate the relationship between the algebra of tensor categories and the topology of framed 3-manifolds. On the one hand, tensor categories with certain algebraic properties determine topological invariants. We prove that fusion categories of nonzero global dimension are 3-dualizable, and therefore provide 3-dimensional 3-framed local field theories. We also show that all finite tensor categories are 2-dualizable, and yield categorified 2-dimensional 3-framed local field theories. On the other hand, topological properties of 3-framed manifolds determine algebraic equations among functors of tensor categories. We show that the 1-dimensional loop bordism, which exhibits a single full rotation, acts as the double dual autofunctor of a tensor category. We prove that the 2-dimensional belt-trick bordism, which unravels a double rotation, operates on any finite tensor category, and therefore supplies a trivialization of the quadruple dual. This approach produces a quadruple-dual theorem for suitably dualizable objects in any symmetric monoidal 3-category. There is furthermore a correspondence between algebraic structures on tensor categories and homotopy fixed point structures, which in turn provide structured field theories; we describe the expected connection between pivotal tensor categories and combed fixed point structures, and between spherical tensor categories and oriented fixed point structures.