Abstract

We solve a number of questions pertaining to the dynamics of linear operators on Hilbert spaces, sometimes by using Baire category arguments and sometimes by constructing explicit examples. In particular, we prove the following results.

(i) A typical hypercyclic operator is not topologically mixing, has no eigenvalues and admits no non-trivial invariant measure, but is densely distributionally chaotic.

(ii) A typical upper-triangular operator with coefficients of modulus 1 on the diagonal is ergodic in the Gaussian sense, whereas a typical operator of the form "diagonal with coefficients of modulus 1 on the diagonal plus backward unilateral weighted shift" is ergodic but has only countably many unimodular eigenvalues; in particular, it is ergodic but not ergodic in the Gaussian sense.

(iii) There exist Hilbert space operators which are chaotic and $\mathcal{U}$-frequently hypercyclic but not frequently hypercyclic, Hilbert space operators which are chaotic and frequently hypercyclic but not ergodic, and Hilbert space operators which are chaotic and topologically mixing but not $\mathcal{U}$-frequently hypercyclic.

We complement our results by investigating the descriptive complexity of some natural classes of operators defined by dynamical properties.