

# Contents

Chapter 1. Introduction	1
1. General overview	1
2. Background and notations	5
2.1. Transitivity, mixing and weak mixing	5
2.2. Chaos	5
2.3. Ergodic-theoretic properties	6
2.4. Frequent hypercyclicity and $\mathcal{U}$ -frequent hypercyclicity	7
2.5. Properties related to eigenvalues	8
2.6. Distributional chaos	8
2.7. The parameter $c(T)$	9
2.8. A last notation	9
3. Organization of the monograph	9
Chapter 2. Typical properties of hypercyclic operators	13
1. The strong and strong* topologies	13
1.1. Why SOT and SOT*?	15
2. How to prove density results	16
3. Construction of mixing operators, and density of $\text{G-MIX}(\mathcal{H})$	17
4. Topological weak mixing and topological mixing	21
4.1. Some illustrations	23
5. Hypercyclic operators without eigenvalues	25
6. Hypercyclic operators without invariant measures	27
7. Densely distributionally chaotic operators	30
8. Summary	34
Chapter 3. Descriptive set-theoretic issues	35
1. Complexity of the families $\text{TMIX}_M(\mathcal{H})$ , $\text{CH}_M(\mathcal{H})$ , $\text{UFHC}_M(\mathcal{H}) \cap \text{CH}_M(\mathcal{H})$ and $\text{UFHC}_M(\mathcal{H})$	35
1.1. Complexity of $\text{TMIX}_M(\mathcal{H})$	35
1.2. Complexity of $\text{CH}_M(\mathcal{H})$	37
1.3. Complexity of $\text{UFHC}_M(\mathcal{H})$ and $\text{UFHC}_M(\mathcal{H}) \cap \text{CH}_M(\mathcal{H})$	41
2. Some non-Borel sets in $\mathfrak{B}_M(\mathcal{H})$	47
Chapter 4. Ergodicity for upper-triangular operators	53
1. Definitions and setting	53
2. Perfect spanning is typical	54
3. Ergodicity vs ergodicity in the Gaussian sense	57
3.1. Statement of the main result	57
3.2. Preliminaries on the eigenvalues of $T_{\lambda, \omega}$	58
3.3. Comeagerness of $\mathcal{D}_M$	59

3.4. An auxiliary result	59
3.5. Comeagerness of $\mathcal{E}_M$	61
3.6. Some comments on ergodicity and unimodular eigenvalues	63
4. Additional remarks	63
4.1. Some natural questions	63
4.2. More on the operators $T_{\lambda, \omega}$	64
Chapter 5. Periodic points at the service of hypercyclicity	69
1. Precompact orbits and topological mixing	69
2. Uniform recurrence and topological weak mixing	70
3. A criterion for $\mathcal{U}$ -frequent hypercyclicity	74
3.1. Uniform recurrence, almost periodic points and $\mathcal{U}$ -frequent hypercyclicity	76
3.2. More about $\mathcal{U}$ -frequent hypercyclicity and $c(T)$	80
4. A criterion for frequent hypercyclicity	84
4.1. Link with the Operator Specification Property	90
Chapter 6. Operators of C-type and of $C_+$ -type	93
1. Operators of C-type: basic facts	93
2. Operators of $C_+$ -type: how to be FHC or UFHC	96
3. Operators of C-type: how <i>not</i> to be FHC or UFHC	99
3.1. The main criterion, in abstract form	100
3.2. How to check the assumptions	104
4. Operators of C-type: how to be mixing or not mixing	109
Chapter 7. Explicit counterexamples	113
1. Summary	113
2. Operators of $C_{+,1}$ -type: FHC does not imply ergodic	113
2.1. How to be FHC or UFHC	114
2.2. A word about the OSP	116
2.3. FHC but not ergodic	117
3. Operators of $C_{+,2}$ -type: UFHC does not imply FHC	119
3.1. How to be FHC or UFHC	119
3.2. UFHC but not FHC	121
4. Operators of $C_2$ -type: chaos plus mixing do not imply UFHC	122
4.1. How to be topologically mixing	122
4.2. How not to be UFHC	123
4.3. Chaotic and mixing operators which are not UFHC	125
5. Chaos plus mixing plus FHC do not imply ergodicity	127
6. C-type operators with few eigenvalues	128
7. $C_{+,1}$ -type with many eigenvalues	133
8. Infinite direct sums of frequently hypercyclic operators	135
Chapter 8. A few questions	137
Short list of abbreviations	139
Bibliography	141