Abstract

We describe a method, based on the theory of Macdonald–Koornwinder polynomials, for proving bounded Littlewood identities. Our approach provides an alternative to Macdonald's partial fraction technique and results in the first examples of bounded Littlewood identities for Macdonald polynomials. These identities, which take the form of decomposition formulas for Macdonald polynomials of type (R, S) in terms of ordinary Macdonald polynomials, are q, t-analogues of known branching formulas for characters of the symplectic, orthogonal and special orthogonal groups. In the classical limit, our method implies that MacMahon's famous ex-conjecture for the generating function of symmetric plane partitions in a box follows from the identification of $\big(\mathrm{GL}(n, \mathbb{R}), \mathrm{O}(n)\big)$ as a Gelfand pair. As further applications, we obtain combinatorial formulas for characters of affine Lie algebras; Rogers–Ramanujan identities for affine Lie algebras, complementing recent results of Griffin et al.; and quadratic transformation formulas for Kaneko–Macdonald-type basic hypergeometric series.