Abstract

Some scales of spaces of ultra-differentiable functions are introduced, having good stability properties with respect to infinitely many derivatives and compositions. They are well-suited for solving non-linear functional equations by means of hard implicit function theorems. They comprise Gevrey functions and thus, as a limiting case, analytic functions. Using majorizing series, we manage to characterize them in terms of a real sequence M bounding the growth of derivatives.

In this functional setting, we prove two fundamental results of Hamiltonian perturbation theory: the invariant torus theorem, where the invariant torus remains ultra-differentiable under the assumption that its frequency satisfies some arithmetic condition which we call BR_M, and which generalizes the Bruno-Rüssmann condition; and Nekhoroshev's theorem, where the stability time depends on the ultra-differentiable class of the pertubation, through the same sequence M. Our proof uses periodic averaging, while a substitute for the analyticity width allows us to bypass analytic smoothing.

We also prove converse statements on the destruction of invariant tori and on the existence of diffusing orbits with ultra-differentiable perturbations, by respectively mimicking a construction of Bessi (in the analytic category) and Marco-Sauzin (in the Gevrey non-analytic category). When the perturbation space satisfies some additional condition (we then call it *matching*), we manage to narrow the gap between stability hypotheses (e.g. the BR_M condition) and instability hypotheses, thus circumbscribing the stability threshold.

The formulas relating the growth M of derivatives of the perturbation on the one hand, and the arithmetics of robust frequencies or the stability time on the other hand, bring light to the competition between stability properties of nearly integrable systems and the distance to integrability. Due to our method of proof using width of regularity as a regularizing parameter, these formulas are closer to optimal as the the regularity tends to analyticity.