Abstract

We introduce (pre-)Galois and cleft monoidal cowreaths. Generalizing a result of Schneider, to any pre-Galois cowreath we associate a pair of adjoint functors $L \dashv R$ and give necessary and sufficient conditions for the adjunction to be an equivalence of categories. Inspired by the work of Doi we also give sufficient conditions for $L \dashv R$ to be an equivalence, and consequently conditions under which a fundamental structure theorem for entwined modules over monoidal cowreaths holds. We show that a cowreath is cleft if and only if it is Galois and has the normal basis property; this generalizes a result concerning Hopf cleft extensions due to Doi and Takeuchi. Furthermore, we show that the cleft cowreaths are in a one to one correspondence with what we call cleft wreaths. The latter are wreaths in the sense of Lack and Street, equipped with two additional morphisms satisfying some compatibility relations. Note that, in general, the algebras defined by cleft wreaths cannot be identified to (generalized) crossed product algebras, as they were defined by Doi and Takeuchi, and Blattner, Cohen and Montgomery. This becomes more transparent when we apply our theory to cowreaths defined by actions and coactions of a quasi-Hopf algebra, monoidal entwining structures and ν-Doi-Hopf structures, respectively. In particular, we obtain that some constructions of Brzeziński and Schauenburg produce examples of cleft wreaths, and therefore of cleft cowreaths, too.