Abstract

For a connected smooth projective curve X of genus g, global sections of any line bundle L with $\deg(L) \geq 2g+1$ give an embedding of the curve into projective space. We consider an analogous statement for a Berkovich skeleton in nonarchimedean geometry: We replace projective space by tropical projective space, and an embedding by a homeomorphism onto its image preserving integral structures (or equivalently, since X is a curve, an isometry), which is called a faithful tropicalization.

Let K be an algebraically closed field which is complete with respect to a nontrivial nonarchimedean value. Suppose that X is defined over K and has genus $g \geq 2$ and that Γ is a skeleton (that is allowed to have ends) of the analytification X^{an} of X in the sense of Berkovich. We show that if $\deg(L) \geq 3g-1$, then global sections of L give a faithful tropicalization of Γ into tropical projective space.

As an application, when Y is a suitable affine curve, we describe the analytification Y^{an} as the limit of tropicalizations of an effectively bounded degree.