Abstract

We contribute to the classification of Hopf algebras with finite Gelfand-Kirillov dimension, GKdim for short, through the study of Nichols algebras over abelian groups. We deal first with braided vector spaces over $\mathbb{Z}$ with the generator acting as a single Jordan block and show that the corresponding Nichols algebra has finite GKdim if and only if the size of the block is 2 and the eigenvalue is ± 1; when this is 1, we recover the quantum Jordan plane. We consider next a class of braided vector spaces that are direct sums of blocks and points that contains those of diagonal type. We conjecture that a Nichols algebra of diagonal type has finite GKdim if and only if the corresponding generalized root system is finite. Assuming the validity of this conjecture, we classify all braided vector spaces in the mentioned class whose Nichols algebra has finite GKdim. Consequently we present several new examples of Nichols algebras with finite GKdim, including two not in the class alluded to above. We determine which among these Nichols algebras are domains.