

## Abstract

We construct a Goodwillie tower of categories which interpolates between the category of pointed spaces and the category of spectra. This tower of categories refines the Goodwillie tower of the identity functor in a precise sense. More generally, we construct such a tower for a large class of  $\infty$ -categories  $\mathcal{C}$  and classify such Goodwillie towers in terms of the derivatives of the identity functor of  $\mathcal{C}$ . As a particular application we show how this provides a model for the homotopy theory of simply-connected spaces in terms of coalgebras in spectra with Tate diagonals. Our classification of Goodwillie towers simplifies considerably in settings where the Tate cohomology of the symmetric groups vanishes. As an example we apply our methods to rational homotopy theory. Another application identifies the homotopy theory of  $p$ -local spaces with homotopy groups in a certain finite range with the homotopy theory of certain algebras over Ching's spectral version of the Lie operad. This is a close analogue of Quillen's results on rational homotopy.