

## Abstract

We shall establish the core of singular integral theory and pseudodifferential calculus over the archetypal algebras of noncommutative geometry: quantum forms of Euclidean spaces and tori. Our results go beyond Connes' pseudodifferential calculus for rotation algebras, thanks to a new form of Calderón-Zygmund theory over these spaces which crucially incorporates nonconvolution kernels. We deduce  $L_p$ -boundedness and Sobolev  $p$ -estimates for regular, exotic and forbidden symbols in the expected ranks. In the  $L_2$  level both Calderón-Vaillancourt and Bourdaud theorems for exotic and forbidden symbols are also generalized to the quantum setting. As a basic application of our methods, we prove  $L_p$ -regularity of solutions for elliptic PDEs.