

Abstract

We show that all versions of Heegaard Floer homology, link Floer homology, and sutured Floer homology are natural. That is, they assign concrete groups to each based 3-manifold, based link, and balanced sutured manifold, respectively. Furthermore, we functorially assign isomorphisms to (based) diffeomorphisms, and show that this assignment is isotopy invariant.

The proof relies on finding a simple generating set for the fundamental group of the “space of Heegaard diagrams,” and then showing that Heegaard Floer homology has no monodromy around these generators. In fact, this allows us to give sufficient conditions for an arbitrary invariant of multi-pointed Heegaard diagrams to descend to a natural invariant of 3-manifolds, links, or sutured manifolds.