Abstract

In this paper, we establish a necessary and sufficient condition for the existence and regularity of the density of the solution to a semilinear stochastic (fractional) heat equation with measure-valued initial conditions. Under a mild cone condition for the diffusion coefficient, we establish the smooth joint density at multiple points. The tool we use is Malliavin calculus. The main ingredient is to prove that the solutions to a related stochastic partial differential equation have negative moments of all orders. Because we cannot prove $u(t,x) \in \mathbb{D}^\infty$ for measure-valued initial data, we need a localized version of Malliavin calculus. Furthermore, we prove that the (joint) density is strictly positive in the interior of the support of the law, where we allow both measure-valued initial data and unbounded diffusion coefficient. The criteria introduced by Bally and Pardoux are no longer applicable for the *parabolic Anderson model*. We have extended their criteria to a localized version. Our general framework includes the parabolic Anderson model as a special case.