Abstract

Many geometric and analytic properties of sets hinge on the properties of elliptic measure, notoriously missing for sets of higher co-dimension. The aim of this manuscript is to develop a version of elliptic theory, associated to a linear PDE, which ultimately yields a notion analogous to that of the harmonic measure, for sets of codimension higher than 1.

To this end, we turn to degenerate elliptic equations. Let $\Gamma \subset \mathbb{R}^n$ be an Ahlfors regular set of dimension $d < n-1$ (not necessarily integer) and $\Omega = \mathbb{R}^n \setminus \Gamma$. Let $L = -\operatorname{div} A\nabla$ be a degenerate elliptic operator with measurable coefficients such that the ellipticity constants of the matrix A are bounded from above and below by a multiple of $\operatorname{dist}(\cdot, \Gamma)^{d+1-n}$. We define weak solutions; prove trace and extension theorems in suitable weighted Sobolev spaces; establish the maximum principle, De Giorgi-Nash-Moser estimates, the Harnack inequality, the Hölder continuity of solutions (inside and at the boundary). We define the Green function and provide the basic set of pointwise and/or L^p estimates for the Green function and for its gradient. With this at hand, we define harmonic measure associated to L, establish its doubling property, non-degeneracy, change-of-the-pole formulas, and, finally, the comparison principle for local solutions.

In another article to appear, we will prove that when Γ is the graph of a Lipschitz function with small Lipschitz constant, we can find an elliptic operator L

2010 *Mathematics Subject Classification.* Primary 31B05, 35J25; Secondary 28A75, 28A78 , 35J70, 42B20, 42B25, 42B37.

Key words and phrases. Harmonic measure, boundary of co-dimension higher than 1, trace theorem, extension theorem, degenerate elliptic operators, maximum principle, Hölder continuity of solutions, de Giorgi-Nash-Moser estimates, Green functions, comparison principle, homogeneous weighted Sobolev spaces.

This research was supported in part by Fondation Jacques Hadamard and by CNRS. The first author was supported in part by the ANR, programme blanc GEOMETRYA ANR-12-BS01-0014, the European Community Marie Curie grant MANET 607643 and H2020 grant GHAIA 777822, and the Simons Collaborations in MSP grant 601941, GD. The third author was supported by the Alfred P. Sloan Fellowship, the NSF INSPIRE Award DMS 1344235, NSF CAREER Award DMS 1220089, the NSF RAISE-TAQ grant DMS 1839077, and the Simons Foundation grant 563916, SM..

We would like to thank the Department of Mathematics at Université Paris-Sud, the Ecole des Mines, and the Mathematical Sciences Research Institute (NSF grant DMS 1440140) for warm hospitality.

Finally, we would like to thank Alano Ancona for stimulating discussions at the early stages of the project and for sharing with us the results of his work.

for which the harmonic measure given here is absolutely continuous with respect to the d-Hausdorff measure on Γ and vice versa. It thus extends Dahlberg's theorem to some sets of codimension higher than 1.