

Abstract

We present several formulae for the large t asymptotics of the Riemann zeta function $\zeta(s)$, $s = \sigma + it$, $0 \leq \sigma \leq 1$, $t > 0$, which are valid to all orders. A particular case of these results coincides with the classical results of Siegel. Using these formulae, we derive explicit representations for the sum $\sum_a^b n^{-s}$ for certain ranges of a and b . In addition, we present precise estimates relating this sum with the sum $\sum_c^d n^{s-1}$ for certain ranges of a, b, c, d . We also study a two-parameter generalization of the Riemann zeta function which we denote by $\Phi(u, v, \beta)$, $u \in \mathbb{C}$, $v \in \mathbb{C}$, $\beta \in \mathbb{R}$. Generalizing the methodology used in the study of $\zeta(s)$, we derive asymptotic formulae for $\Phi(u, v, \beta)$.