

Contents

Chapter 1. Introduction	1
Chapter 2. Notation and Preliminaries	5
Chapter 3. Maximal Subgroups	11
Chapter 4. Maximal Subgroups and Subspace Stabilizers	19
Chapter 5. Blueprint Theorems for Semisimple Elements	27
5.1. Preliminary Results	28
5.2. Determination of the Bounds for the Minimal Module	29
5.3. Consequences for Maximal Subgroups	35
Chapter 6. Unipotent and Semisimple Elements	37
6.1. Actions of Unipotent Elements	37
6.2. Blueprints and Element Orders	39
6.3. Blueprints inside Subgroups of Type A_1	41
6.4. Traces of Modules for PGL_2	43
6.5. The Graph Automorphism of F_4	45
6.6. Rank-1 Subalgebras of the Lie Algebra	45
Chapter 7. Modules for SL_2	49
7.1. Modules for $\mathrm{SL}_2(2^a)$	49
7.2. Modules for $\mathrm{SL}_2(3^a)$	54
7.3. Modules for $\mathrm{SL}_2(p)$	57
7.4. Modules for $\mathrm{SL}_2(p^a)$ for $p \geq 5$ and $a > 1$	62
Chapter 8. Some PSL_2 s inside E_6 in Characteristic 3	67
Chapter 9. Proof of the Theorems: Strategy	71
Chapter 10. The Proof for F_4	75
10.1. Characteristic 2	75
10.2. Characteristic 3	78
10.3. Characteristic At Least 5	86
Chapter 11. The Proof for E_6	89
11.1. Characteristic 2	89
11.2. Characteristic 3	90
11.3. Characteristic At Least 5	90
Chapter 12. The Proof for E_7 in Characteristic 2	97

Chapter 13. The Proof for E_7 in Odd Characteristic: PSL_2 Embedding	115
13.1. Characteristic 3	115
13.2. Characteristic At Least 5	117
Chapter 14. The Proof for E_7 in Odd Characteristic: SL_2 Embedding	125
14.1. Characteristic 3	125
14.2. Characteristic At Least 5	130
Appendix A. Actions of Maximal Positive-Dimensional Subgroups on Minimal and Adjoint Modules	149
Appendix B. Traces of Small-Order Semisimple Elements	151
Bibliography	153