

# Contents

Chapter 1. Introduction	1
1.1. What we knew	1
1.2. What we wanted	4
1.3. What we did	6
1.4. How we proved it	9
1.5. Classification of non-separable structures up to bi-embeddability	10
1.6. Organization of the paper, or: How (not) to read this paper	12
1.7. Annotated content	13
Chapter 2. Preliminaries and notation	17
2.1. Basic notions	17
2.2. Choice and determinacy	18
2.3. Cardinality	19
2.4. Algebras of sets	19
2.5. Descriptive set theory	21
2.6. Trees and reductions	23
Chapter 3. The generalized Cantor space	27
3.1. Basic facts	27
More on $2^\kappa$	29
Chapter 4. Generalized Borel sets	37
4.1. Basic facts	37
4.2. Intermezzo: the projective ordinals	40
More on generalized Borel sets	41
Chapter 5. Generalized Borel functions	45
5.1. Basic facts	45
Further results	47
Chapter 6. The generalized Baire space and Baire category	51
6.1. The generalized Baire space	51
6.2. Baire category	54
Chapter 7. Standard Borel $\kappa$ -spaces, $\kappa$ -analytic quasi-orders, and spaces of codes	59
7.1. $\kappa$ -analytic sets	59
7.2. Spaces of type $\kappa$ and spaces of codes	62
Chapter 8. Infinitary logics and models	69
8.1. Infinitary logics	69

8.2. Some generalizations of the Lopez-Escobar theorem	73
Chapter 9. $\kappa$ -Souslin sets	81
9.1. Basic facts	81
9.2. More on Souslin sets and Souslin cardinals	83
9.3. Souslin sets and cardinals in models with choice	85
9.4. Souslin sets and cardinals in models of determinacy	88
Chapter 10. The main construction	95
10.1. The combinatorial trees $\mathbb{G}_0$ and $\mathbb{G}_1$	96
10.2. The combinatorial trees $\mathbb{G}_S$	101
Chapter 11. Completeness	105
11.1. Faithful representations of $\kappa$ -Souslin quasi-orders	105
11.2. The quasi-order $\leq_{\max}$ and the reduction $\Sigma_T$	107
11.3. Reducing $\leq_{\max}^\kappa$ to $\sqsubset_{CT}^\kappa$	108
11.4. Some absoluteness results	110
Chapter 12. Invariant universality	113
12.1. An $\mathcal{L}_{\kappa+\kappa}$ -sentence $\Psi$ describing the structures $\mathbb{G}_S$ .	113
12.2. A classification of the structures in $\text{Mod}_\Psi^\kappa$ up to isomorphism	120
12.3. The invariant universality of $\sqsubset_{CT}^\kappa$	124
12.4. More absoluteness results	127
Chapter 13. An alternative approach	131
13.1. Completeness	132
13.2. Invariant universality	133
Chapter 14. Definable cardinality and reducibility	143
14.1. Topological complexity	144
14.2. Absolutely definable reducibilities	147
14.3. Reducibilities in an inner model	152
Chapter 15. Some applications	155
15.1. $\Sigma_2^1$ quasi-orders	155
15.2. Projective quasi-orders	157
15.3. More complex quasi-orders in models of determinacy	159
15.4. $L(\mathbb{R})$ -reducibility	160
Chapter 16. Further completeness results	163
16.1. Representing arbitrary partial orders as embeddability relations	163
16.2. Other model theoretic examples	166
16.3. Isometry and isometric embeddability between complete metric spaces of density character $\kappa$	169
16.4. Linear isometry and linear isometric embeddability between Banach spaces of density $\kappa$	175
Further results on the classification of nonseparable metric and Banach spaces	177
Indexes	181
Concepts	181

CONTENTS

v

Symbols	182
Bibliography	185