Abstract

We discuss the combinatorics of decorated Dyck paths and decorated parallelogram polyominoes, extending to the decorated case the main results of both Haglund ("A proof of the q,t-Schröder conjecture", 2004) and Aval et al. ("Statistics on parallelogram polyominoes and a q,t-analogue of the Narayana numbers", 2014). This settles in particular the cases $\langle \cdot, e_{n-d}h_d \rangle$ and $\langle \cdot, h_{n-d}h_d \rangle$ of the Delta conjecture of Haglund, Remmel and Wilson ("The delta conjecture", 2018). Along the way, we introduce some new statistics, formulate some new conjectures, prove some new identities of symmetric functions, and answer a few open problems in the literature (e.g., from Aval, Bergeron and Garsia [2015], Haglund, Remmel and Wilson [2018], and Zabrocki [2019]). The main technical tool is a new identity in the theory of Macdonald polynomials that extends a theorem of Haglund in "A proof of the q,t-Schröder conjecture" (2004).