Abstract

This is the second in a pair of works which study small disturbances to the plane, periodic 3D Couette flow in the incompressible Navier-Stokes equations at high Reynolds number $\mathbf{Re}$. In this work, we show that there is constant $0 < c_0 \ll 1$, independent of $\mathbf{Re}$, such that sufficiently regular disturbances of size $\epsilon \lesssim \mathbf{Re}^{-2/3-\delta}$ for any $\delta > 0$ exist at least until $t = c_0\epsilon^{-1}$ and in general evolve to be $O(c_0)$ due to the lift-up effect. Further, after times $t \gtrsim \mathbf{Re}^{1/3}$, the streamwise dependence of the solution is rapidly diminished by a mixing-enhanced dissipation effect and the solution is attracted back to the class of "2.5 dimensional" streamwise-independent solutions (sometimes referred to as "streaks"). The largest of these streaks are expected to eventually undergo a secondary instability at $t \approx \epsilon^{-1}$. Hence, our work strongly suggests, for *all* (sufficiently regular) initial data, the genericity of the "lift-up effect $\Rightarrow$ streak growth $\Rightarrow$ streak breakdown" scenario for turbulent transition of the 3D Couette flow near the threshold of stability forwarded in the applied mathematics and physics literature.