

Contents

Chapter 1. Introduction	1
1.1. Linear behavior and streaks	4
1.2. Statement of main results	6
1.3. Notations and conventions	8
Acknowledgments	10
Chapter 2. Outline of the proof	11
2.1. Summary and weakly nonlinear heuristics	11
2.2. Choice of the norms	12
2.3. Instantaneous regularization and continuation of solutions	14
2.4. Q^i formulation, the coordinate transformation, and some key cancellations	15
2.5. The toy model and design of the norms	18
2.6. Design of the norms based on the toy model	21
2.7. Main energy estimates	23
Chapter 3. Regularization and continuation	27
Chapter 4. Multiplier and paraproduct tools	29
4.1. Basic inequalities regarding the multipliers	30
4.2. Paraproducts and related notations	41
4.3. Product lemmas and a few immediate consequences	42
Chapter 5. High norm estimate on Q^2	45
5.1. Zero frequencies	46
5.2. Non-zero frequencies	51
Chapter 6. High norm estimate on Q^3	63
6.1. Zero frequencies	64
6.2. Non-zero frequencies	66
Chapter 7. High norm estimate on Q_0^1	81
7.1. Transport nonlinearity	82
7.2. Nonlinear stretching	82
7.3. Forcing from non-zero frequencies	82
7.4. Dissipation error terms	84
Chapter 8. High norm estimate on Q_{\neq}^1	85
8.1. Linear stretching term $LS1$	85
8.2. Lift-up effect term LU	86
8.3. Linear pressure term $LP1$	87

8.4. Nonlinear pressure NLP	87
8.5. Nonlinear stretching NLS	87
8.6. Transport nonlinearity \mathcal{T}	90
8.7. Dissipation error terms \mathcal{D}	92
Chapter 9. Coordinate system controls	93
9.1. High norm estimate on g	93
9.2. Low norm estimate on g	96
9.3. Long time, high norm estimate on C^i	96
9.4. Shorter time, high norm estimate on C^i	98
9.5. Low norm estimate on C	100
Chapter 10. Enhanced dissipation estimates	101
10.1. Enhanced dissipation of Q^3	101
10.2. Enhanced dissipation of Q^2	107
10.3. Enhanced dissipation of Q^1	111
Chapter 11. Sobolev estimates	115
11.1. Improvement of (2.45c) and (2.45b)	115
11.2. Improvement of (2.45a)	116
Appendix A. Fourier analysis conventions, elementary inequalities, and Gevrey spaces	117
Appendix B. Some details regarding the coordinate transform	119
Appendix C. Definition and analysis of the norms	121
C.1. Definition and analysis of w	121
C.2. The design and analysis of w_L	124
Appendix D. Elliptic estimates	125
D.1. Lossy estimates	125
D.2. Precision lemmas	126
Bibliography	131