Abstract

In 1975 Figari, Høegh-Krohn and Nappi constructed the $\mathscr{P}(\varphi)_2$ model on the de Sitter space. Here we complement their work with new results, which connect this model to various areas of mathematics. In particular,

i.) we discuss the causal structure of de Sitter space and the induces representations of the Lorentz group. We show that the UIRs of $SO_0(1,2)$ for both the principal and the complementary series can be formulated on Hilbert spaces whose functions are supported on a Cauchy surface. We describe the free classical dynamical system in both its covariant and canonical form, and present the associated quantum one-particle KMS structures in the sense of Kay (1985). Furthermore, we discuss the localisation properties of one-particle wave functions and how these properties are inherited by the algebras of local observables.

ii.) we describe the relations between the modular objects (in the sense of Tomita-Takesaki theory) associated to wedge algebras and the representations of the Lorentz group. We connect the representations of SO(1,2) to unitary representations of $SO(3)$ on the Euclidean sphere, and discuss how the $\mathscr{P}(\varphi)_2$ interaction can be represented by a rotation invariant vector in the Euclidean Fock space. We present a novel Osterwalder-Schrader reconstruction theorem, which shows that physical *infrared problems* are absent on de Sitter space. As shown in Figari, Høegh-Krohn, and Nappi (1975), the ultraviolet problems are resolved just like on flat Minkowski space. We state the Haag–Kastler axioms for the $\mathscr{P}(\varphi)_2$ model and we explain how the *generators* of the boosts and the rotations for the interacting quantum field theory arise from the *stress-energy tensor*. Finally, we

2020 *Mathematics Subject Classification.* Primary 81T05, 81T08, 81T20; Secondary 22E43, 47L90.

Key words and phrases. De Sitter space, unitary irreducible representations, Fourier–Helgason transformation, (constructive) quantum field theory.

The second author is grateful to IHES, Bures-sur-Yvette, France, for generous hospitality offered during a visit in late 2014 and CERN, Geneva, were final revisions of the manuscript were undertaken during a visit in June 2016. The second and the third author were partially supported by Conselho Nacional de Desenvolvimento Cientifico e Tecnológico (CNPq), in particular by Pro-jeto Universal 438435/2018-4 and by the São Paulo Research Foundation (FAPESP); in particular, the second author was supported by a visiting professorship in 2012-2013 and by Projeto Regular 2018/09613-9 and the third author was supported by FAPESP grant number 2014/24522-9.

show that the interacting quantum fields satisfy the *equations of motion* in their covariant form.

In summary, we argue that the de Sitter $\mathscr{P}(\varphi)_2$ model is the simplest and most explicit relativistic quantum field theory, which satisfies basic expectations, like covariance, particle creation, stability and finite speed of propagation.