Abstract

We investigate the role played by curve singularity germs in the enumeration of inflection points in families of curves acquiring singular members. Let $N \geq 2$, and consider an isolated complete intersection curve singularity germ $f\colon (\mathbb{C}^N, 0) \to (\mathbb{C}^{N-1}, 0)$. We define a numerical function $m \mapsto \mathrm{AD}^m_{(2)}(f)$ that naturally arises when counting m^{th}-order weight-2 inflection points with ramification sequence $(0, \dots, 0, 2)$ in a 1-parameter family of curves acquiring the singularity $f = 0$, and we compute $\mathrm{AD}^m_{(2)}(f)$ for several interesting families of pairs (f, m). In particular, for a node defined by $f\colon (x, y) \mapsto xy$, we prove that $\mathrm{AD}^m_{(2)}(xy) = \binom{m+1}{4}$, and we deduce as a corollary that $\mathrm{AD}^m_{(2)}(f) \geq (\mathrm{mult}_0\, \Delta_f) \cdot \binom{m+1}{4}$ for any f, where $\mathrm{mult}_0\, \Delta_f$ is the multiplicity of the discriminant Δ_f at the origin in the deformation space. Significantly, we prove that the function $m \mapsto \mathrm{AD}^m_{(2)}(f) - (\mathrm{mult}_0\, \Delta_f) \cdot \binom{m+1}{4}$ is an analytic invariant measuring how much the singularity "counts as" an inflection point. We prove similar results for weight-2 inflection points with ramification sequence $(0, \dots, 0, 1, 1)$ and for weight-1 inflection points, and we apply our results to solve a number of related enumerative problems.