Abstract

The box-ball system (BBS), introduced by Takahashi and Satsuma in 1990, is a cellular automaton that exhibits solitonic behaviour. In this article, we study the BBS when started from a random two-sided infinite particle configuration. For such a model, Ferrari et al. recently showed the invariance in distribution of Bernoulli product measures with density strictly less than $\frac{1}{2}$, and gave a soliton decomposition for invariant measures more generally. We study the BBS dynamics using the transformation of a nearest neighbour path encoding of the particle configuration given by 'reflection in the past maximum', which was famously shown by Pitman to connect Brownian motion and a three-dimensional Bessel process. We use this to characterise the set of configurations for which the dynamics are well-defined and reversible (i.e. can be inverted) for all times. The techniques developed to understand the deterministic dynamics are subsequently applied to study the evolution of the BBS from a random initial configuration. Specifically, we give simple sufficient conditions for random initial conditions to be invariant in distribution under the BBS dynamics, which we check in several natural examples, and also investigate the ergodicity of the relevant transformation. Furthermore, we analyse various probabilistic properties of the BBS that are commonly studied for interacting particle systems, such as the asymptotic behavior of the integrated current of particles and of a tagged particle. Finally, for Bernoulli product measures with parameter $p \uparrow \frac{1}{2}$ (which may be considered the 'high density' regime), the path encoding we consider has a natural scaling limit, which motivates the introduction of a new continuous version of the BBS that we believe will be of independent interest as a dynamical system.