

Abstract

The weight multiplicities of finite dimensional simple Lie algebras can be computed individually using various methods. Still, it is hard to derive explicit closed formulas. Similarly, explicit closed formulas for the multiplicities of maximal weights of affine Kac–Moody algebras are not known in most cases. In this paper, we study weight multiplicities for both finite and affine cases of classical types for certain infinite families of highest weights modules. We introduce new classes of Young tableaux, called the *(spin) rigid tableaux*, and prove that they are equinumerous to the weight multiplicities of the highest weight modules under our consideration. These new classes of Young tableaux arise from crystal basis elements for dominant maximal weights of the integrable highest weight modules over affine Kac–Moody algebras. By applying combinatorics of tableaux such as the Robinson–Schensted algorithm and new insertion schemes, and using integrals over orthogonal groups, we reveal hidden structures in the sets of weight multiplicities and obtain explicit closed formulas for the weight multiplicities. In particular we show that some special families of weight multiplicities form the Pascal, Catalan, Motzkin, Riordan and Bessel triangles.