

# Contents

Introduction	1
Acknowledgments	5
Chapter 1. Affine Kac–Moody algebras	7
1.1. Preliminaries	7
1.2. Quantum affine algebras	10
Chapter 2. Crystals and Young walls	11
2.1. Crystals	11
2.2. Connection to finite type	12
2.3. Young walls for level 1 representations	13
2.4. Higher level representations	18
Chapter 3. Young tableaux and almost even tableaux	23
3.1. Young tableaux	23
3.2. Tensor products of Young walls	25
3.3. Some families of Young tableaux	25
Chapter 4. Lattice paths and triangular arrays	27
4.1. Motzkin triangle	27
4.2. Riordan triangle	28
4.3. Catalan triangle	30
4.4. Pascal Triangle	31
Chapter 5. Dominant maximal weights	33
5.1. Type $A_{n-1}^{(1)}$	33
5.2. Type $B_n^{(1)}$	34
5.3. Type $C_n^{(1)}$	42
5.4. Type $D_n^{(1)}$	43
5.5. Type $A_{2n-1}^{(2)}$	45
5.6. Type $A_{2n}^{(2)}$	46
5.7. Type $D_{n+1}^{(2)}$	47
5.8. Classification of staircase dominant maximal weights	48
Chapter 6. Weight multiplicities and (spin) rigid Young tableaux	49
6.1. Case $\text{smax}_{\mathfrak{B}}^+(\Lambda k)$	50
6.2. Case $\text{smax}_{\mathfrak{D}}^+(\Lambda k)$	52
Chapter 7. Level 2 weight multiplicities: Catalan and Pascal triangles	59
7.1. Classical realizations	59

7.2.	Insertion of a box	60
7.3.	Case $\text{smax}_{\mathfrak{B}}^+(\Lambda 2)$	61
7.4.	Case $\text{smax}_{\mathfrak{D}}^+(\Lambda 2)$	63
Chapter 8.	Level 3 weight multiplicities: Motzkin and Riordan triangles	65
8.1.	Proof by the RS algorithm	66
8.2.	Proof by insertion scheme	70
Chapter 9.	Some level $k$ weight multiplicities when $k \rightarrow \infty$ : Bessel triangle	75
9.1.	The limit of $ \mathfrak{D}_m^{(k)} $ when $k \rightarrow \infty$	75
9.2.	The limit of $ \mathfrak{B}_m^{(k)} $ when $k \rightarrow \infty$	76
9.3.	The limit of $ \mathfrak{S}_m^{(k)} $ when $k \rightarrow \infty$	77
Chapter 10.	Standard Young tableaux with a fixed number of rows of odd length	79
10.1.	The cardinality of $\mathcal{S}_m^{(k,t)}$ for $0 \leq t \leq k \leq 5$	79
10.2.	Traces of orthogonal matrices	83
10.3.	Evaluation of integrals	84
Bibliography		87