Abstract

We consider the wave maps problem with domain $\mathbb{R}^{2+1}$ and target $\mathbb{S}^2$ in the 1-equivariant, topological degree one setting. In this setting, we recall that the soliton is a harmonic map from $\mathbb{R}^2$ to $\mathbb{S}^2$, with polar angle equal to $Q_1(r) = 2\arctan(r)$. By applying the scaling symmetry of the equation, $Q_\lambda(r) = Q_1(r\lambda)$ is also a harmonic map, and the family of all such Q_λ are the unique minimizers of the harmonic map energy among finite energy, 1-equivariant, topological degree one maps. In this work, we construct infinite time blowup solutions along the Q_λ family. More precisely, for $b > 0$, and for all $\lambda_{0,0,b} \in C^\infty([100,\infty))$ satisfying, for some $C_l, C_{m,k} > 0$,

$$\frac{C_l}{\log^b(t)} \leq \lambda_{0,0,b}(t) \leq \frac{C_m}{\log^b(t)}, \quad |\lambda_{0,0,b}^{(k)}(t)| \leq \frac{C_{m,k}}{t^k \log^{b+1}(t)}, k \geq 1 \quad t \geq 100$$

there exists a wave map with the following properties. If u_b denotes the polar angle of the wave map into $\mathbb{S}^2$, we have

$$u_b(t,r) = Q_{\frac{1}{\lambda_b(t)}}(r) + v_2(t,r) + v_e(t,r), \quad t \geq T_0$$

where

$$-\partial_{tt} v_2 + \partial_{rr} v_2 + \frac{1}{r}\partial_r v_2 - \frac{v_2}{r^2} = 0$$

$$||\partial_t (Q_{\frac{1}{\lambda_b(t)}} + v_e)||_{L^2(rdr)}^2 + ||\frac{v_e}{r}||_{L^2(rdr)}^2 + ||\partial_r v_e||_{L^2(rdr)}^2 \leq \frac{C}{t^2 \log^{2b}(t)}, \quad t \geq T_0$$

and

$$\lambda_b(t) = \lambda_{0,0,b}(t) + O\left(\frac{1}{\log^b(t)\sqrt{\log(\log(t))}}\right)$$