Abstract

Weighted projective lines, introduced by Geigle and Lenzing in 1987, are important objects in representation theory. They have tilting bundles, whose endomorphism algebras are the canonical algebras introduced by Ringel. The aim of this paper is to study their higher dimensional analogs. First, we introduce a certain class of commutative Gorenstein rings R graded by abelian groups $\mathbb{L}$ of rank 1, which we call Geigle-Lenzing complete intersections. We study the stable category $\underline{\mathsf{CM}}^{\mathbb{L}}R$ of Cohen-Macaulay representations, which coincides with the singularity category $\mathsf{D}_{\mathrm{sg}}^{\mathbb{L}}(R)$. We show that $\underline{\mathsf{CM}}^{\mathbb{L}}R$ is triangle equivalent to $\mathsf{D}^{\mathrm{b}}(\mathsf{mod}A^{\mathrm{CM}})$ for a finite dimensional algebra A^{CM}, which we call the CM-canonical algebra. As an application, we classify the $(R,\mathbb{L})$ that are Cohen-Macaulay finite. We also give sufficient conditions for $(R,\mathbb{L})$ to be d-Cohen-Macaulay finite in the sense of higher Auslander-Reiten theory. Secondly, we study a new class of non-commutative projective schemes in the sense of Artin-Zhang, i.e. the category $\mathsf{coh}\mathbb{X} = \mathsf{mod}^{\mathbb{L}}R/\mathsf{mod}_0^{\mathbb{L}}R$ of coherent sheaves on the Geigle-Lenzing projective space $\mathbb{X}$. Geometrically this is the quotient stack $\mathbb{X} = [X/G]$ for $X = \operatorname{Spec} R \setminus \{R_+\}$ and $G = \operatorname{Spec} k[\mathbb{L}]$. We show that $\mathsf{D}^{\mathrm{b}}(\mathsf{coh}\mathbb{X})$ is triangle equivalent to $\mathsf{D}^{\mathrm{b}}(\mathsf{mod}A^{\mathrm{ca}})$ for a finite dimensional algebra A^{ca}, which we call a d-canonical algebra. We study when $\mathbb{X}$ is d-vector bundle finite, and when $\mathbb{X}$ is derived equivale'nt to a d-representation infinite algebra in the sense of higher Auslander-Reiten theory. Our d-canonical algebras provide a rich source of d-Fano and d-anti-Fano algebras in non-commutative algebraic geometry. We also observe Orlov-type semiorthogonal decompositions of $\mathsf{D}_{\mathrm{sg}}^{\mathbb{L}}(R)$ and $\mathsf{D}^{\mathrm{b}}(\mathsf{coh}\mathbb{X})$.