Abstract

Spiral waves are striking self-organized coherent structures that organize spatio-temporal dynamics in dissipative, spatially extended systems. In this paper, we provide a conceptual approach to various properties of spiral waves. Rather than studying existence in a specific equation, we study properties of spiral waves in general reaction-diffusion systems. We show that many features of spiral waves are robust and to some extent independent of the specific model analyzed. To ac-complish this, we present a suitable analytic framework, spatial radial dynamics, that allows us to rigorously characterize features such as the shape of spiral waves and their eigenfunctions, properties of the linearization, and finite-size effects. We believe that our framework can also be used to study spiral waves further and help analyze bifurcations, as well as provide guidance and predictions for exper-iments and numerical simulations. From a technical point of view, we introduce non-standard function spaces for the well-posedness of the existence problem which allow us to understand properties of spiral waves using dynamical systems tech-niques, in particular exponential dichotomies. Using these pointwise methods, we are able to bring tools from the analysis of one-dimensional coherent structures such as fronts and pulses to bear on these inherently two-dimensional defects.

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