

Abstract

We study continuity, and lack thereof, of thermodynamical properties for one-dimensional dynamical systems. Under quite general hypotheses, the free energy is shown to be almost upper-semicontinuous: some normalised component of a limit measure will have free energy at least that of the limit of the free energies. From this, we deduce results concerning existence and continuity of equilibrium states (including statistical stability). Metric entropy, not semicontinuous as a general multimodal map varies, is shown to be upper semicontinuous under an appropriate hypothesis on critical orbits. Equilibrium states vary continuously, under mild hypotheses, as one varies the parameter and the map. We give a general method for constructing induced maps which automatically give strong exponential tail estimates. This also allows us to recover, and further generalise, recent results concerning statistical properties (decay of correlations, etc.). Counterexamples to statistical stability are given which also show sharpness of the main results.