

# Contents

Chapter 1. Introduction	1
Motivational background	1
Aim of this monograph	4
Acknowledgement	8
<b>Part 1. Preliminaries, properties of period functions, and some insights</b>	<b>9</b>
Chapter 2. Notations	11
Chapter 3. Elements from hyperbolic geometry	13
3.1. Models and isometries	13
3.2. Classification of isometries	14
3.3. Cusps, funnels, limit set, and ordinary points	14
3.4. Geodesics, resonances, and the Selberg zeta function	14
3.5. Intervals and rounded neighborhoods	16
Chapter 4. Hecke triangle groups with infinite covolume	19
Chapter 5. Automorphic forms	21
5.1. Funnel forms of different types	21
5.2. Fourier expansion	22
Chapter 6. Principal series	25
6.1. Regularity at infinity	26
6.2. Presheaves and sheaves	26
6.3. Holomorphic extensions	26
Chapter 7. Transfer operators and period functions	29
7.1. Discretizations and transfer operators	29
7.2. Slow transfer operators	32
7.3. Period functions	33
7.4. Real and complex period functions	34
7.5. Fast transfer operators	37
7.6. One-sided averages	39
7.7. Convergence and meromorphic extension of fast transfer operators	40
7.8. Spaces of complex period functions	41
Chapter 8. An intuition and some insights	43
<b>Part 2. Semi-analytic cohomology</b>	<b>47</b>

Chapter 9. Abstract cohomology spaces	49
9.1. Standard group cohomology	49
9.2. Cohomology on an invariant set	49
9.3. Relation to parabolic cohomology spaces	52
Chapter 10. Modules	53
10.1. Modules of semi-analytic functions	53
10.2. Submodules of semi-analytic vectors	54
10.3. Conditions on cocycles	56
10.4. Cohomological interpretation of the singularity condition	57
<b>Part 3. Automorphic forms and cohomology</b>	<b>63</b>
Chapter 11. Invariant eigenfunctions via a group cohomology	67
Chapter 12. Tessellation cohomology	69
12.1. Choice of a tessellation, and cohomology	69
12.2. Relation to group cohomology	72
12.3. Mixed cohomology spaces	74
Chapter 13. Extension of cocycles	77
Chapter 14. Surjectivity I: Boundary germs	87
14.1. Analytic boundary germs and semi-analytic modules	87
14.2. Cohomology classes attached to funnel forms	89
14.3. Representatives of boundary germs	91
Chapter 15. Surjectivity II: From cocycles to funnel forms	93
15.1. From a cocycle to an invariant eigenfunction	93
15.2. A cocycle on an orbit of ordinary points	97
15.3. Isomorphisms	103
Chapter 16. Relation between cohomology spaces	105
Chapter 17. Proof of Theorem D	109
From funnel forms to cocycle classes on the invariant set	109
From cocycle classes on to funnel forms	110
Proof of Theorem D	111
<b>Part 4. Transfer operators and cohomology</b>	<b>113</b>
Chapter 18. The map from functions to cocycles	117
Chapter 19. Real period functions and semi-analytic cocycles	121
Chapter 20. Complex period functions and semi-analytic cohomology	127
Chapter 21. Proof of Theorem E	135
<b>Part 5. Proofs of Theorems A and B, and a recapitulation</b>	<b>137</b>
<b>Part 6. Parity</b>	<b>141</b>

Chapter 22. The triangle group in the projective general linear group	145
22.1. Two actions of the projective general linear group	145
22.2. The triangle group	146
Chapter 23. Odd and even funnel forms, cocycles, and period functions	147
23.1. Odd and even funnel forms	147
23.2. Odd and even cocycles	148
23.3. Odd and even period functions	150
Chapter 24. Isomorphisms with parity	153
<b>Part 7. Complements and outlook</b>	<b>157</b>
Chapter 25. Fredholm determinant of the fast transfer operator	159
Chapter 26. Outlook	163
Bibliography	165
Index of terminology	169
List of notations	171