Contents

Chapter 1. Introduction 1
Acknowledgments 6

Chapter 2. Measure-theoretic preliminaries 7
2.1. Some basic notions 7
2.2. Couplings 12
2.3. Closed properties in a coupling space 18
2.4. Localization 19
2.5. Conditional independence in set lattices 23
2.6. Idempotent couplings 25

Chapter 3. Cubic couplings 33
3.1. Conditional independence of simplicial sets 37
3.2. Tricubes 39
3.3. U^d-convolutions and U^d-seminorms associated with a cubic coupling 40
3.4. Fourier σ-algebras 42
3.5. Properties of U^d-convolutions 45
3.6. Topologization of cubic couplings 49
3.7. Continuous U^n-convolutions 53
3.8. Topological nilspace factors of X 53

Chapter 4. The structure theorem for cubic couplings 61
4.1. Verifying the ergodicity and composition axioms 63
4.2. Complete dependence of corner couplings 64
4.3. Convolution neighbourhoods 67
4.4. Construction of the coupling Υ. 68
4.5. Verifying the corner-completion axiom 73

Chapter 5. On characteristic factors associated with nilpotent group actions 77

Chapter 6. On cubic exchangeability 85

Chapter 7. Limits of functions on compact nilspaces 91

Appendix A. Background results from measure theory 93

Bibliography 99