Abstract

This monograph introduces a framework for genuine proper equivariant stable homotopy theory for Lie groups. The adjective 'proper' alludes to the feature that equivalences are tested on compact subgroups, and that the objects are built from equivariant cells with compact isotropy groups; the adjective 'genuine' indicates that the theory comes with appropriate transfers and Wirthmüller isomorphisms, and the resulting equivariant cohomology theories support the analog of an $RO(G)$-grading.

Our model for genuine proper G-equivariant stable homotopy theory is the category of orthogonal G-spectra; the equivalences are those morphisms that induce isomorphisms of equivariant stable homotopy groups for all compact subgroups of G. This class of π_*-isomorphisms is part of a symmetric monoidal stable model structure, and the associated tensor triangulated homotopy category is compactly generated. Consequently, every orthogonal G-spectrum represents an equivariant cohomology theory on the category of G-spaces. These represented cohomology theories are designed to only depend on the 'proper G-homotopy type', tested by fixed points under all compact subgroups.

An important special case of our theory are infinite discrete groups. For these, our genuine equivariant theory is related to finiteness properties in the sense of geometric group theory; for example, the G-sphere spectrum is a compact object in our triangulated equivariant homotopy category if the universal space for proper

G-actions has a finite G-CW-model. For discrete groups, the represented equivariant cohomology theories on finite proper G-CW-complexes admit a more explicit description in terms of parameterized equivariant homotopy theory, suitably stabilized by G-vector bundles. Via this description, we can identify the previously defined G-cohomology theories of equivariant stable cohomotopy and equivariant K-theory as cohomology theories represented by specific orthogonal G-spectra.