Abstract

We present a general method to extend results on Hilbert space operators to the Banach space setting by representing certain sets of Banach space operators Γ on a Hilbert space. Our assumption on Γ is expressed in terms of α-boundedness for a Euclidean structure α on the underlying Banach space X. This notion is originally motivated by $\mathcal{R}$- or γ-boundedness of sets of operators, but for example any operator ideal from the Euclidean space ℓ_n^2 to X defines such a structure. Therefore, our method is quite flexible. Conversely we show that Γ has to be α-bounded for some Euclidean structure α to be representable on a Hilbert space.

By choosing the Euclidean structure α accordingly, we get a unified and more general approach to the Kwapień–Maurey factorization theorem and the factorization theory of Maurey, Nikišin and Rubio de Francia. This leads to an improved version of the Banach function space-valued extension theorem of Rubio de Francia and a quantitative proof of the boundedness of the lattice Hardy–Littlewood maximal operator. Furthermore, we use these Euclidean structures to build vector-valued function spaces. These enjoy the nice property that any bounded operator on L^2 extends to a bounded operator on these vector-valued function spaces, which is in stark contrast to the extension problem for Bochner spaces. With these spaces we define an interpolation method, which has formulations modelled after both the real and the complex interpolation method.

Using our representation theorem, we prove a transference principle for sectorial operators on a Banach space, enabling us to extend Hilbert space results for sectorial operators to the Banach space setting. We for example extend and refine the known theory based on $\mathcal{R}$-boundedness for the joint and operator-valued H^∞-calculus.

Moreover, we extend the classical characterization of the boundedness of the H^∞-calculus on Hilbert spaces in terms of BIP, square functions and dilations to the Banach space setting. Furthermore we establish, via the H^∞-calculus, a version of Littlewood–Paley theory and associated spaces of fractional smoothness for a rather large class of sectorial operators. Our abstract setup allows us to reduce assumptions on the geometry of X, such as (co)type and UMD. We conclude with some sophisticated counterexamples for sectorial operators, with as a highlight the construction of a sectorial operator of angle 0 on a closed subspace of L^p for $1 < p < \infty$ with a bounded H^∞-calculus with optimal angle $\omega_{H^\infty}(A) > 0$.