

# Contents

Introduction	1
Chapter 1. Euclidean structures and $\alpha$ -bounded operator families	9
1.1. Euclidean structures	10
1.2. $\alpha$ -bounded operator families	18
1.3. The representation of $\alpha$ -bounded operator families on a Hilbert space	23
1.4. The equivalence of $\alpha$ -boundedness and $C^*$ -boundedness	25
Chapter 2. Factorization of $\alpha$ -bounded operator families	29
2.1. Factorization of $\gamma$ - and $\pi_2$ -bounded operator families	29
2.2. $\alpha$ -bounded operator families on Banach function spaces	34
2.3. Factorization of $\ell^2$ -bounded operator families through $L^2(S, w)$	38
2.4. Banach function space-valued extensions of operators	41
Chapter 3. Vector-valued function spaces and interpolation	53
3.1. The spaces $\alpha(H, X)$ and $\alpha(S; X)$	53
3.2. Function space properties of $\alpha(S; X)$	60
3.3. The $\alpha$ -interpolation method	65
3.4. A comparison with real and complex interpolation	69
Chapter 4. Sectorial operators and $H^\infty$ -calculus	77
4.1. The Dunford calculus	78
4.2. (Almost) $\alpha$ -sectorial operators	80
4.3. $\alpha$ -bounded $H^\infty$ -calculus	84
4.4. Operator-valued and joint $H^\infty$ -calculus	89
4.5. $\alpha$ -bounded imaginary powers	97
Chapter 5. Sectorial operators and generalized square functions	103
5.1. Generalized square function estimates	104
5.2. Dilations of sectorial operators	111
5.3. A scale of generalized square function spaces	115
5.4. Generalized square function spaces without almost $\alpha$ -sectoriality	122
Chapter 6. Some counterexamples	129
6.1. Schauder multiplier operators	129
6.2. Sectorial operators which are not almost $\alpha$ -sectorial	135
6.3. Almost $\alpha$ -sectorial operators which are not $\alpha$ -sectorial	137
6.4. Sectorial operators with $\omega_{H^\infty}(A) > \omega(A)$	141
Bibliography	149