Abstract

We study embeddings of groups of Lie type H in characteristic p into exceptional algebraic groups $\mathbf{G}$ of the same characteristic. We exclude the case where H is of type PSL_2. A subgroup of $\mathbf{G}$ is *Lie primitive* if it is not contained in any proper, positive-dimensional subgroup of $\mathbf{G}$.

With a few possible exceptions, we prove that there are no Lie primitive subgroups H in $\mathbf{G}$, with the conditions on H and $\mathbf{G}$ given above. The exceptions are for H one of $\mathrm{PSL}_3(3)$, $\mathrm{PSU}_3(3)$, $\mathrm{PSL}_3(4)$, $\mathrm{PSU}_3(4)$, $\mathrm{PSU}_3(8)$, $\mathrm{PSU}_4(2)$, $\mathrm{PSp}_4(2)'$ and ${}^2B_2(8)$, and $\mathbf{G}$ of type E_8. No examples are known of such Lie primitive embeddings.

We prove a slightly stronger result, including stability under automorphisms of $\mathbf{G}$. This has the consequence that, with the same exceptions, any almost simple group with socle H, that is maximal inside an almost simple exceptional group of Lie type F_4, E_6, ${}^2E_6$, E_7 and E_8, is the fixed points under the Frobenius map of a corresponding maximal closed subgroup inside the algebraic group.

The proof uses a combination of representation-theoretic, algebraic group-theoretic, and computational means.