Abstract

Leclerc and Zelevinsky, motivated by the study of quasi-commuting quantum flag minors, introduced the notions of *strongly separated* and *weakly separated* collections. These notions are closely related to the theory of *cluster algebras,* to the combinatorics of the *double Bruhat cells,* and to the *totally positive Grassmannian.*

A key feature, called the *purity phenomenon,* is that every maximal by inclusion strongly (resp., weakly) separated collection of subsets in $[n]$ has the same cardinality.

In this paper, we extend these notions and define *$\mathcal{M}$-separated collections* for any oriented matroid $\mathcal{M}$.

We show that maximal by size $\mathcal{M}$-separated collections are in bijection with fine zonotopal tilings (if $\mathcal{M}$ is a realizable oriented matroid), or with one-element liftings of $\mathcal{M}$ in general position (for an arbitrary oriented matroid).

We introduce the class of *pure oriented matroids* for which the purity phenomenon holds: an oriented matroid $\mathcal{M}$ is pure if $\mathcal{M}$-separated collections form a pure simplicial complex, i.e., any maximal by inclusion $\mathcal{M}$-separated collection is also maximal by size.

We pay closer attention to several special classes of oriented matroids: oriented matroids of rank 3, graphical oriented matroids, and uniform oriented matroids. We classify pure oriented matroids in these cases. An oriented matroid of rank 3 is pure if and only if it is a *positroid* (up to reorienting and relabeling its ground set). A graphical oriented matroid is pure if and only if its underlying graph is an *outerplanar graph,* that is, a subgraph of a triangulation of an n-gon.

We give a simple conjectural characterization of pure oriented matroids by forbidden minors and prove it for the above classes of matroids (rank 3, graphical, uniform).