

Abstract

The aim of this monograph is to study the global existence of solutions to a coupled wave-Klein-Gordon system in space dimension two when initial data are small smooth and mildly decaying at infinity. Some physical models strictly related to general relativity have shown the importance of studying such systems but very few results are known at present in low space dimension. We study here a model two-dimensional system, in which the nonlinearity writes in terms of “null forms”, and show the global existence of small solutions. Our goal is to prove some energy estimates on the solution when a certain number of Klainerman vector fields is acting on it, and some optimal uniform estimates. The former ones are obtained using systematically quasilinear normal forms, in their para-differential version; the latter ones are recovered by deducing a new coupled system of a transport equation and an ordinary differential equation from the starting PDE system by means of a semiclassical micro-local analysis of the problem. We expect the strategy developed here to be robust enough to enable us, in the future, to treat the case of the most general nonlinearities.