Abstract

Various coordinate rings of varieties appearing in the theory of Poisson Lie groups and Poisson homogeneous spaces belong to the large, axiomatically defined class of symmetric Poisson nilpotent algebras, e.g. coordinate rings of Schubert cells for symmetrizable Kac–Moody groups, affine charts of Bott-Samelson varieties, coordinate rings of double Bruhat cells (in the last case after a localization). We prove that every symmetric Poisson nilpotent algebra satisfying a mild condition on certain scalars is canonically isomorphic to a cluster algebra which coincides with the corresponding upper cluster algebra, without additional localizations by frozen variables. The constructed cluster structure is compatible with the Poisson structure in the sense of Gekhtman, Shapiro and Vainshtein. All Poisson nilpotent algebras are proved to be equivariant Poisson Unique Factorization Domains. Their seeds are constructed from sequences of Poisson-prime elements for chains of Poisson UFDs; mutation matrices are effectively determined from linear systems in terms of the underlying Poisson structure. Uniqueness, existence, mutation, and other properties are established for these sequences of Poisson-prime elements.