Abstract

We establish global-in-time frequency localized local smoothing estimates for Schrödinger equations on hyperbolic space $\mathbb{H}^d$, $d \geq 2$. In the presence of symmetric first and zeroth order potentials, which are possibly time-dependent, possibly large, and have sufficiently fast polynomial decay, these estimates are proved up to a localized lower order error. Then in the time-independent case, we show that a spectral condition (namely, absence of threshold resonances) implies the full local smoothing estimates (without any error), after projecting to the continuous spectrum. In the process, as a means to localize in frequency, we develop a general Littlewood–Paley machinery on $\mathbb{H}^d$ based on the heat flow. Our results and techniques are motivated by applications to the problem of stability of solitary waves to nonlinear Schrödinger-type equations on $\mathbb{H}^d$. Specifically, some of the estimates established in this paper play a crucial role in the authors' proof of the nonlinear asymptotic stability of harmonic maps under the Schrödinger maps evolution on the hyperbolic plane.

As a testament of the robustness of approach, which is based on the positive commutator method and a heat flow based Littlewood-Paley theory, we also show that the main results are stable under small time-dependent perturbations, including polynomially decaying second order ones, and small lower order nonsymmetric perturbations.