

Contents

Chapter 1. Introduction	1
1.1. Statements of the main results	2
1.2. History and related works	11
1.3. Structure of the paper	12
Acknowledgments	13
Chapter 2. Preliminaries, Function Spaces and Multipliers	15
2.1. Notation	15
2.2. Geometry of the domain and polar coordinates	15
2.3. L^p and Sobolev spaces on \mathbb{H}^d	17
2.4. Basic comparisons of LE_s and LE_{low}	18
2.5. Time-independent and auxiliary function spaces	18
2.6. The Main Multiplier Identity	22
2.7. Slowly varying functions α and the multiplier	24
Chapter 3. Overview of the scheme and outline of the paper	27
3.1. Outline of the proof of Theorems 1.5 and 1.15	27
3.2. Outline of the proof of Theorems 1.11 and 1.16	32
3.3. Outline of the remainder of the article	34
Chapter 4. Regularity theory of the heat flow: Littlewood–Paley theory	35
4.1. Localized Parabolic Regularity	36
4.2. Estimating lower order terms in LE^*	46
4.3. Embeddings of the local smoothing spaces	58
4.4. Estimates controlling frequency localized error in $L^2(\mathbb{R} \times \{r \leq R\})$	61
4.5. Boundedness of the multiplier Q	63
4.6. Estimates needed to handle F	69
4.7. Estimates needed to handle $H_{1,\text{o.t.}}$	71
4.8. Estimates to commute $\mathbf{P}_s, \mathbf{P}_{\geq s_0}$ with H_{prin}	82
4.9. Estimates needed to handle $\bar{H}_{\text{prin}} + \Delta$	91
4.10. An elliptic regularity estimate	94
4.11. Estimates needed to handle time-dependent perturbations	100
Chapter 5. Smoothing for Low Frequencies	107
Chapter 6. Smoothing for High Frequencies	115
Chapter 7. Transitioning Estimate and Proof of Theorems 1.5 and 1.15	127
Chapter 8. Local smoothing estimate in the stationary, symmetric case and its perturbations	135

8.1. Key multiplier identities	135
8.2. Extraction of a resonance	138
8.3. Absence of embedded resonances	147
8.4. Equivalent formulations of the threshold nonresonance condition	151
8.5. Perturbation of symmetric, stationary case	155
Chapter 9. Proof of Corollaries 1.18 and 1.19	157
9.1. Local smoothing for the projections P_s and $P_{\geq s}$	157
9.2. Strichartz estimates	159
9.3. Combined Functional Framework	160
Bibliography	163