Abstract

In this paper, we study the dynamics of fluids in porous media governed by Darcy's law: the Muskat problem. We consider the setting of two immiscible fluids of different densities and viscosities under the influence of gravity in which one fluid is completely surrounded by the other. This setting is gravity unstable because along a portion of the interface, the denser fluid must be above the other. Surprisingly, even without capillarity, the circle-shaped bubble is a steady state solution moving with vertical constant velocity determined by the density jump between the fluids. Taking advantage of our discovery of this steady state, we are able to prove global in time existence and uniqueness of dynamic bubbles of nearly circular shapes under the influence of surface tension. We prove this global existence result for low regularity initial data. Moreover, we prove that these solutions are instantly analytic and decay exponentially fast in time to the circle.