

## Abstract

We develop a theory of average sizes of kernels of generic matrices with support constraints defined in terms of graphs and hypergraphs. We apply this theory to study unipotent groups associated with graphs. In particular, we establish strong uniformity results pertaining to zeta functions enumerating conjugacy classes of these groups. We deduce that the numbers of conjugacy classes of  $\mathbf{F}_q$ -points of the groups under consideration depend polynomially on  $q$ . Our approach combines group theory, graph theory, toric geometry, and  $p$ -adic integration.

Our uniformity results are in line with a conjecture of Higman on the numbers of conjugacy classes of unitriangular matrix groups. Our findings are, however, in stark contrast to related results by Belkale and Brosnan on the numbers of generic symmetric matrices of given rank associated with graphs.