Abstract

Let K be an algebraically closed field of characteristic zero, and let G be a connected reductive algebraic group over K. We address the problem of classifying triples (G, H, V), where H is a proper connected subgroup of G, and V is a finite-dimensional irreducible G-module such that the restriction of V to H is multiplicity-free – that is, each of its composition factors appears with multiplicity 1. A great deal of classical work, going back to Dynkin, Howe, Kac, Stembridge, Weyl and others, and also more recent work of the authors, can be set in this context. In this paper we determine all such triples in the case where H and G are both simple algebraic groups of type A, and H is embedded irreducibly in G. While there are a number of interesting familes of such triples (G, H, V), the possibilities for the highest weights of the representations defining the embeddings $H < G$ and $G < GL(V)$ are very restricted. For example, apart from two exceptional cases, both weights can only have support on at most two fundamental weights; and in many of the examples, one or other of the weights corresponds to the alternating or symmetric square of the natural module for either G or H.