

Contents

Chapter 1. Introduction	1
Acknowledgment	5
Chapter 2. Notation	7
Chapter 3. Level set-up	9
Chapter 4. Results from the Literature	13
4.1. Littlewood-Richardson theorem	13
4.2. Decomposing the tensor square	15
4.3. Results of Stembridge and Cavallin	16
Chapter 5. Composition Factors In Levels	19
5.1. The main result on levels	19
5.2. Proof of Theorem 5.1.1	22
5.3. Levels for $X = A_2$	28
5.4. Y -levels	30
5.5. Method of Proof - Level Analysis	30
Chapter 6. Multiplicity-free families	33
6.1. Restrictions of SL_n representations to SO_n	33
6.2. Table 1.1 configurations	34
6.2.1. Weights $c\omega_i + \omega_{i+1}$ and $\omega_i + c\omega_{i+1}$	34
6.2.2. Weights $c\omega_1 + \omega_i$	36
6.2.3. Weights $\omega_1 + c\omega_i$	39
6.3. Remaining Table 1.1 configurations	41
6.4. Table 1.2 configurations	48
6.5. Table 1.3 configurations	52
6.6. Table 1.4 configurations	54
6.6.1. Embedding $X = A_3, \delta = \omega_2$	54
6.6.2. $X = A_4, \delta = \omega_2$	66
6.6.3. Remaining Table 1.4 configurations	67
Chapter 7. Initial Lemmas	69
7.1. Summands of Tensor Products	69
7.2. Some non-MF representations	78
7.2.1. Non-MF modules for $\delta = \omega_2$	78
7.2.2. Non-MF modules for $\delta = 2\omega_1$	83
7.2.3. Non-MF symmetric and wedge squares	89
7.2.4. Low rank cases	92
7.2.5. Tensor products, symmetric and exterior powers	101

7.3. $L(\nu) \geq 2$ results	107
Chapter 8. The case $X = A_2$	113
8.1. Case $\delta = rs$ with $r, s > 0$	113
8.1.1. Preliminaries	113
8.1.2. Proof of Theorem 8.1.1	115
8.2. Case $\delta = r0$	121
8.2.1. Case $r = 2$	121
8.2.2. General case $\delta = r\omega_1, r \geq 3$	125
Chapter 9. The case $\delta = r\omega_k$ with $r, k \geq 2$	137
9.1. Case $l > 2$	139
9.2. Case $l = 2$.	146
Chapter 10. The case $\delta = r\omega_1, r \geq 2$	155
10.1. The case $\delta = 2\omega_1$	155
10.1.1. Proof of Theorem 10.1.1	155
10.2. The case $\delta = r\omega_1, r \geq 3$	166
10.2.1. Proof of Theorem 10.2.1	166
Chapter 11. The case $\delta = \omega_i$ with $i \geq 3$	179
11.1. The case where $i < \frac{l+2}{2}$	179
11.2. The case where $i = \frac{l+2}{2}$	181
11.2.1. The case where $\mu^1 \neq 0$	182
11.2.2. The case where $\mu^1 = 0$	183
11.2.3. The case $i = 3, l = 4$	184
Chapter 12. The case $\delta = \omega_2$	189
12.1. $X = A_3, \delta = \omega_2$	189
12.2. $X = A_4, \delta = \omega_2$	193
12.2.1. The case where $\mu^1 = 0$	194
12.2.2. The case where $\mu^1 \neq 0$	204
12.3. $X = A_{l+1}$ with $l \geq 4, \delta = \omega_2$	207
Chapter 13. The case $\delta = \omega_1 + \omega_{l+1}$	217
Chapter 14. Proof of Theorem 1, Part I: $V_{C^i}(\mu^i)$ is usually trivial	223
14.1. Proof of Theorem 14.1	225
14.2. Proof of Theorem 14.2	235
Chapter 15. Proof of Theorem 1, Part II: μ^0 is not inner	239
Chapter 16. Proof of Theorem 1, Part III: $\langle \lambda, \gamma \rangle = 0$	245
Chapter 17. Proof of Theorem 1, Part IV: Completion	253
17.1. Proof of Theorem 17.1	253
17.1.1. The case where $\mu^0 \neq 0, \mu^k = 0$	253
17.1.2. The case where $\mu^0 \neq 0, \mu^k \neq 0$	254
17.1.3. The case where $\mu^0 = 0, \mu^k \neq 0$	257
17.2. Proof of Theorem 17.2: case $a \geq 3$	259
17.3. Proof of Theorem 17.2: case $a = 2$	263

Bibliography

267